+, -, x, and ÷
with Whole Numbers

+ - x ÷

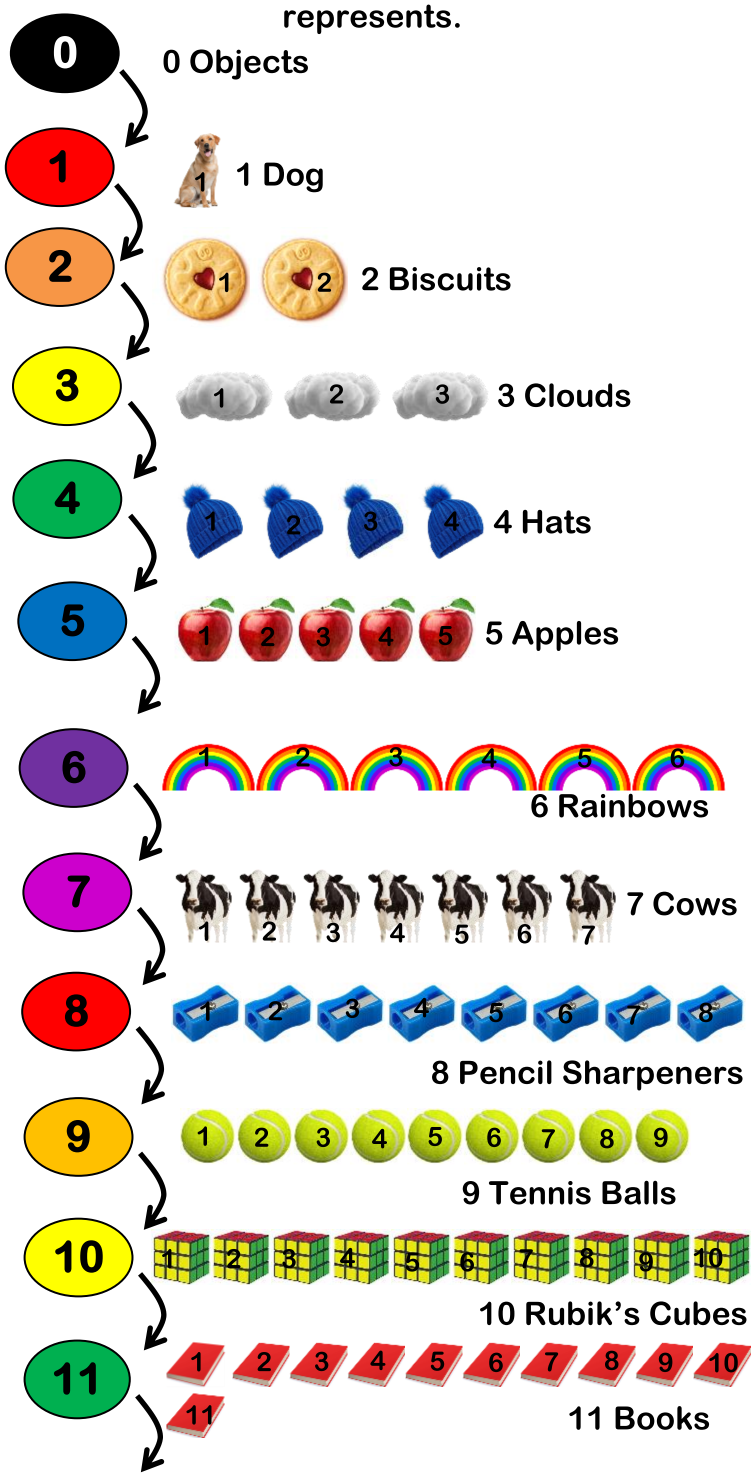
Super Topic Ladder

Step 1) Count up to 20.

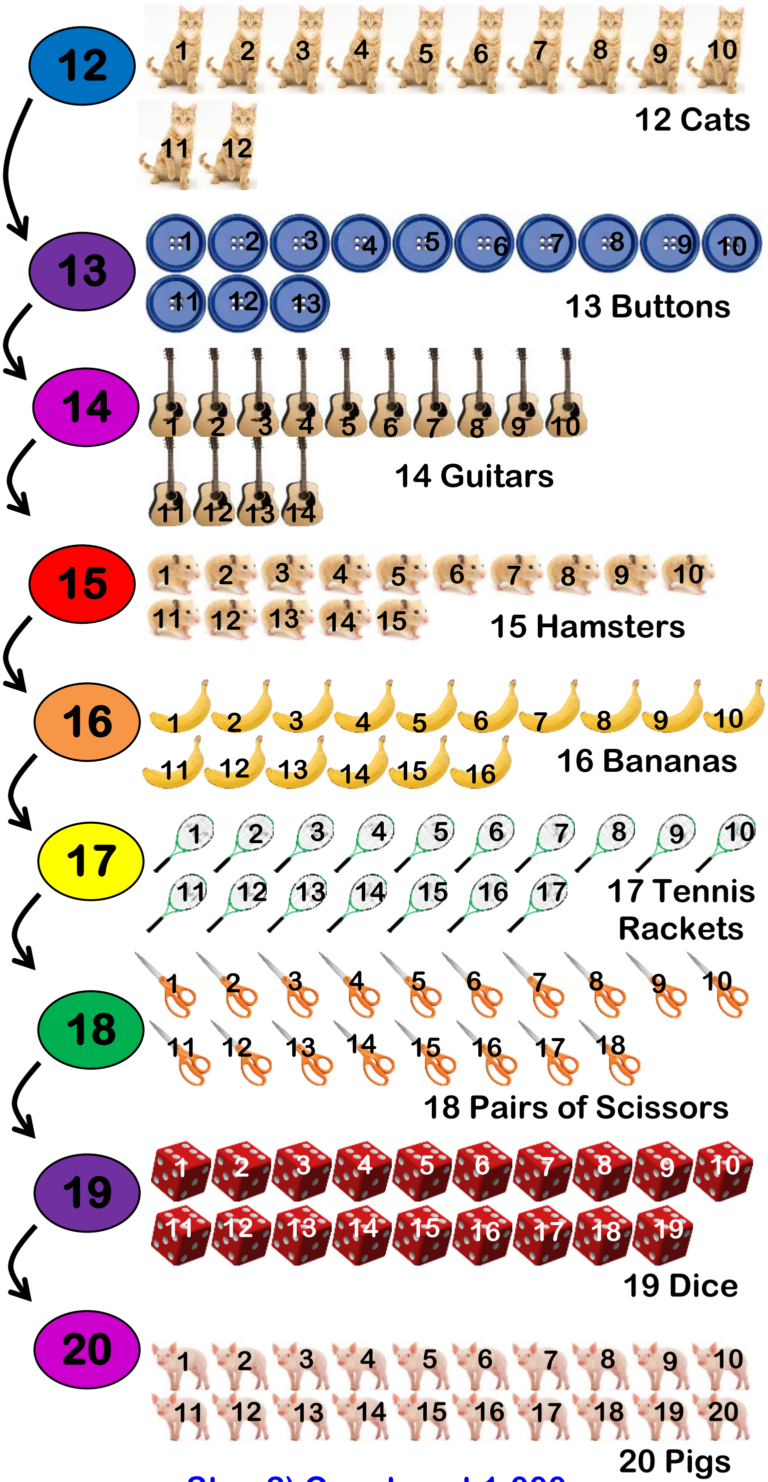
Key Words: "Digit" The numbers 0 to 9
"Unit" is a digit in the 1st column
"Ten" is 10 lots of a unit (2nd column)

With the numbers 1 to 20, you can do most day to day maths to do with knowing how many things there are.
You need to know the order of the numbers, like stepping stones. Once you can count them in the right order, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, you can learn how to count groups of objects.

Here we will put the numbers in order on rainbow coloured stepping stones, and next to that we will draw that number of objects.
For each number we need to know it's place in the order of numbers, and how many objects it represents.



You may notice with 11 that we haven't continued with 11 objects in a row, but have done 10 in a row, and put the 11th on a second line. This is because 10 is a very special number in the decimal system (from Latin word "decem" meaning 10). In the decimal system we organise all our numbers into groups of 10.



Step 2) Count past 1,000

Key Words: "One Hundred" is 10 lots of 10 (3rd column)

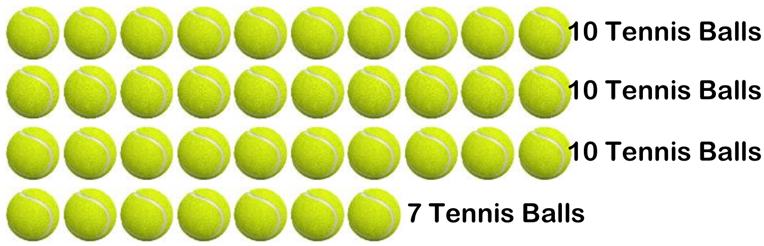
"One Thousand" is 10 lots of 100 (4th column)

After 20, the pattern continues, with a unit 0, 1, 2, 3, 4, 5, 6, 7, 8 0r 9, next to the number of tens. 3 tens is called thirty (30) for example, & with 7 units this would be thirty seven (37).

Here's the numbers 1 to 99, in order. (Grouped in tens)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Let's make 37 tennis balls.
37 = 30 + 7 = 3 tens & 7 units



When you get to 99 objects, you run out of tens and units. 9 tens is called ninety. Then for 10 tens, we have a special number called 100, said one hundred. It goes in the 3rd column of digits. You can build up huge numbers with groups of hundreds represented in this 3rd column.

Let's make 237 rectangles

237 = 200 + 30 + 7 = 2 hundreds + 3 tens + 7 units

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

100 Rectangles (10 Rows of 10)

2 x 100 is 200 Rectangles

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

100 Rectangles (10 Rows of 10)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37			

3 rows of 10 is 30 Rectangles

A row of 7 Rectangles

So 237 rectangles in total!

3,456
= 3,000 + 400 + 50 + 6
= three thousand, four hundred & fifty six

Step 3) + up to 10

Key Words: "Add" Collect together two groups of objects & count as one group.
"Sum" What you get when you add numbers (eg the sum of 2 & 3 is 5)
"Complement (to 10)" The number that pairs to a sum of 10 eg2 is the complement (to 10) of 8

We can add pairs of numbers together and count the total.

2 cats + 3 cats = 5 cats

In general

2 things + 3 things = 5 things

We just say 2 + 3 = 5

As we have seen the number 10 is reeeeeaaaaaaly important. It is super useful to know pairs of numbers that add up to 10.

1	2	3	4	5	6	7	8	9	10

0 + 10 = 10 = 10 + 0

1	2	3	4	5	6	7	8	9	1

9 + 1 = 10 = 1 + 9

1	2	3	4	5	6	7	8	1	2

8 + 2 = 10 = 2 + 8

1	2	3	4	5	6	7	1	2	3

7 + 3 = 10 = 3 + 7

1	2	3	4	5	6	1	2	3	4

6 + 4 = 10 = 4 + 6

1	2	3	4	5	1	2	3	4	5

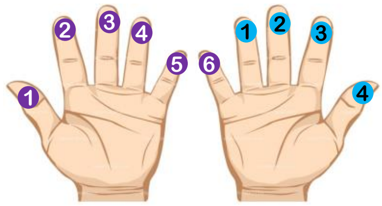
5 + 5 = 10

You can also do this on your 10 fingers.

Take, for example

1	2	3	4	5	6	1	2	3	4

6 + 4 = 10 = 4 + 6



Though this technique is useful, after some practice we get a sense of the pairs and just feel the pairs of numbers that add up to 10.

10
= 0 + 10
= 1 + 9
= 2 + 8
= 3 + 7
= 4 + 6
= 5 + 5
= 6 + 4
= 7 + 3
= 8 + 2
= 9 + 1
= 10 + 0

We also learn that every number can be split into pairs of numbers in different ways.

Let's try with 7.

1	2	3	4	5	6	7

0 + 7 = 7 = 7 + 0

1	2	3	4	5	6	1

6 + 1 = 7 = 1 + 6

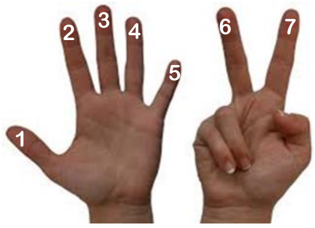
1	2	3	4	5	1	2

5 + 2 = 7 = 2 + 5

1	2	3	4	1	2	3

4 + 3 = 7 = 3 + 4

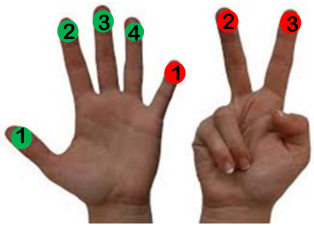
We can also do this on our hands, by folding down three fingers.



With example

1	2	3	4	1	2	3

4 + 3 = 7 = 3 + 4



And again eventually this becomes something we have a sense of, a friendliness with 7, and we just know that:

So 7
= 0 + 7
= 1 + 6
= 2 + 6
= 3 + 4
= 4 + 3
= 5 + 2
= 6 + 1
= 7 + 0

There is a difference between memorising and having a feel for.

1 + 1 = 2 we don't know from memory but because we have a sense of it. Can we increase this sense to all pairs of numbers adding up to anything less than 10?

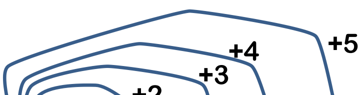
Step 4) + a digit

We have already seen in step 3, that you can add small digits, & find the total, or sum.

Now we will start to add a digit to larger digits and numbers.

Let's start with 7 + 5, this will be bigger than 10.

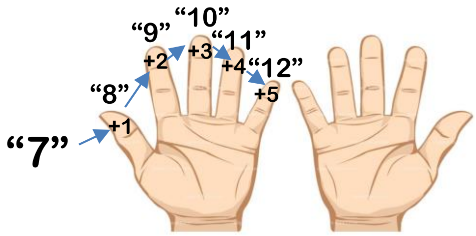
We can just count up 5 from 7, we get to 12.



1	2	3	4	5	6	7	8	9	10	11	12

7 + 5 = 12

Or add 5 by counting up on the fingers.



However, to really get a feel for addition, we have to learn to add past a ten using complements.

We use the fact that 7 + 3 = 10

1	2	3	4	5	6	7	1	2	3
1	2	3	4	5	6	7	8	9	10

And the fact that 3 + 2 = 5

1	2	3	1	2
1	2	3	4	5

7 + 3 = 10							1	2	3	4	5
1	2	3	4	5	6	7	1	2	3	1	2
1	2	3	4	5	6	7	8	9	10	11	12

7 + 5
= 7 + 3 + 2
= 10 + 2
= 12

Either of these methods can be applied when adding a digit to a larger number!

7 + 3 = 10			&	237 + 3 = 240			1	2	3	4	5
1	2	3	4	5	6	7	1	2	3	1	2
231	232	233	234	235	236	237	238	239	240	241	242

237 + 5
=237 + 3 + 2
=240 + 2
= 242

Or even 12,358 + 9
= 12,358 + 2 + 7
= 12,360 + 7
= 12,367

Step 5) - a Digit

Key Words. “Subtract” To remove a number of objects.
“Take Away” meaningful word for subtract

12 – 7
You can think of this as counting back 5 from 12

-5 -4 -3 -2 -1

1	2	3	4	5	6	7	8	9	10	11	12

12 – 5 = 7

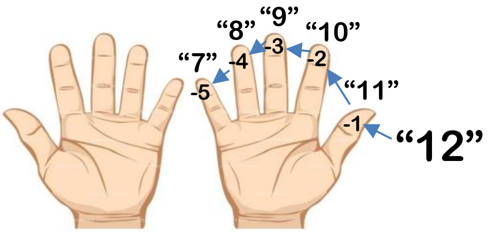
Or you can think of taking 5 things away from 12

-5 -4 -3 -2 -1

1	2	3	4	5	6	7	~~8~~	~~9~~	~~10~~	~~11~~	~~12~~

12 – 5 = 7

And it can also be counted on the fingers.



However, the best method is to count down to 10, and see how many of your 5 you still have left to take away.

7 + 3 = 10, 3 + 2 = 5

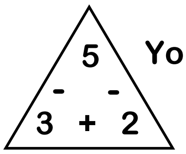
1	2	3	4	5	6	7	1	2	3	1	2
1	2	3	4	5	6	7	8	9	10	11	12

12 – 5
= 12 – 2 – 3
= 10 – 3
= 7

You may have noticed that + & - are opposites. Opposites is the casual word, the formal maths word is inverses.

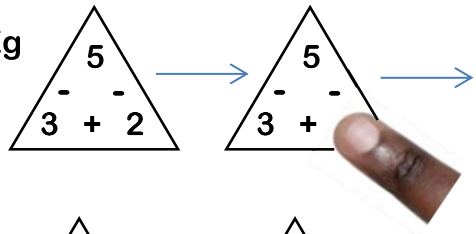
So 2 + 3 = 5
5 - 3 = 2
3 + 2 = 5
5 – 2 = 3

You can organise these linked truths in a triangle:

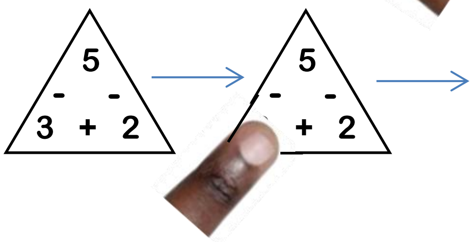


You just cover up the answer!

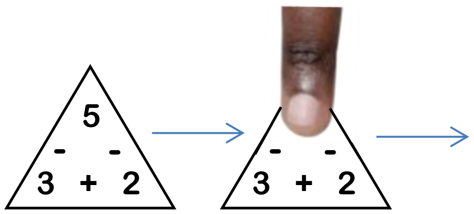
Eg



5 – 3 = 2



5 – 2 = 3



3 + 2 = 5
&
2 + 3 = 5

Step 6) Counting up in 2s, 3s, 4s & 5s, Stepping Stones

When we keep adding the same number, several times, we call it times.
So 3 x 4 (3 times 4)
= 4 + 4 + 4 + 4
= 16

We can think of this visually as jumping along our stepping stones in 2s (for x2), in 3s (for x3), in 4s (for x4) or in 5s (for x5).

Counting in 2s (for x2)

0
1
+ 2 (1st time)
2 — 1 x 2 = 2
3
+ 2 (2nd time)
4 — 2 x 2 = 4
5
+ 2 (3rd time)
6 — 3 x 2 = 6
7
+ 2 (4th time)
8 — 4 x 2 = 8
9
+ 2 (5th time)
10 — 5 x 2 = 10
11
+ 2 (6th time)
12 — 6 x 2 = 12
13
+ 2 (7th time)
14 — 7 x 2 = 14
15
+ 2 (8th time)
16 — 8 x 2 = 16
17
+ 2 (9th time)
18 — 9 x 2 = 18
19
+ 2 (10th time)
20 — 10 x 2 = 20
+ 2

Counting in 3s (for x3)

0
1
2
+ 3 (1st time)
3 — 1 x 3 = 3
4
5
+ 3 (2nd time)
6 — 2 x 3 = 6
7
8
+ 3 (3rd time)
9 — 3 x 3 = 9
10
11
+ 3 (4th time)
12 — 4 x 3 = 12
13
14
+ 3 (5th time)
15 — 5 x 3 = 15
16
17
+ 3 (6th time)
18 — 6 x 3 = 18
19
20
+ 3 (7th time)

Counting in 4s (for x4)

0
1
2
3
+ 4 (1st time)
4 — 1 x 4 = 4
5
6
7
+ 4 (2nd time)
8 — 2 x 4 = 8
9
10
11
+ 4 (3rd time)
12 — 3 x 4 = 12
13
14
15
+ 4 (4th time)
16 — 4 x 4 = 16
17
18
19
+ 4 (5th time)
20 — 5 x 4 = 20

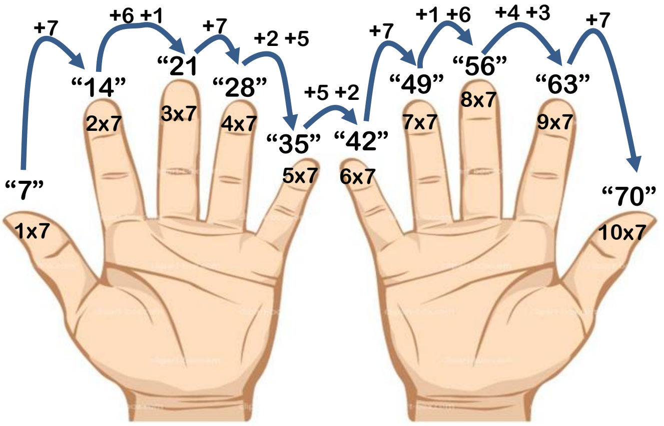
Counting in 5s (for x5)

0
1
2
3
4
+ 5 (1st time)
5 — 1 x 5 = 5
6
7
8
9
+ 5 (2nd time)
10 — 2 x 5 = 10
11
12
13
14
+ 5 (3rd time)
15 — 3 x 5 = 15
16
17
18
19
+ 5 (4th time)
20 — 4 x 5 = 20

Step 7) X & ÷ digits (including 6s, 7s, 8s & 9s).

We can use a method of counting up in 6s, 7s, 8s & 9s too.

Let's look at the 7 times table. We could make some giant leaps on the stepping stones. Or we could count up in 7s on our fingers, using methods from step 4 on adding digits to add 7 each time.



We are using the sum pairs of 7, and the compliments of 10.

Eg from 5x7 = 35 to 6x7 = 42 we do:
+ 5 then + 2

					1	2	3	4	5	6	7
1	2	3	4	5	1	2	3	4	5	1	2
31	32	33	34	35	36	37	38	39	40	41	42

In this way counting up through a times table gives us excellent practice of step 4, and counting down, practice of step 5.

It is called a times table because we often write all the multiples (here of 7) in a table.

The 7 times table.

1x	2x	3x	4x	5x	6x	7x	8x	9x	10x
7	14	21	28	35	42	49	56	63	70

We can put all the times tables together:

	1	2	3	4	5	6	7	8	9	10
1x	1	2	3	4	5	6	7	8	9	10
2x	2	4	6	8	10	12	14	16	18	20
3x	3	6	9	12	15	18	21	24	27	30
4x	4	8	12	16	20	24	28	32	36	40
5x	5	10	15	20	25	30	35	40	45	50
6x	6	12	18	24	30	36	42	48	54	60
7x	7	14	21	28	35	42	49	56	63	70
8x	8	16	24	32	40	48	56	64	72	80
9x	9	18	27	36	45	54	63	72	81	90
10x	10	20	30	40	50	60	70	80	90	100

Let's look at this in a more physical way, as a way of collecting rows of 7 rectangles several times.

1x	1	2	3	4	5	6	7	so 1 x 7 = 7
2x	8	9	10	11	12	13	14	so 2 x 7 = 14
3x	15	16	17	18	19	20	21	so 3 x 7 = 21
4x	22	23	24	25	26	27	28	so 4 x 7 = 28
5x	29	30	31	32	33	34	35	so 5 x 7 = 35
6x	36	37	38	39	40	41	42	so 6 x 7 = 42
7x	43	44	45	46	47	48	49	so 7 x 7 = 49
8x	50	51	52	53	54	55	56	so 8 x 7 = 56
9x	57	58	59	60	61	62	63	so 9 x 7 = 63
10x	64	65	66	67	68	69	70	so 10 x 7 = 70

So in the whole of the large rainbow coloured rectangle we have 70 little rectangles. Each row represents 1 lot of 7.

The thick black lines (eg between the 10th & 11th rectangle, in the second row) highlight where you count up to, if adding up in 7s using compliments of 10, and sum pairs of 7.

Eg from the orange row (after 14 rectangles) you add 6 to make 20, then 1 more to make 21. Adding 6, and adding 1 is like adding 7.

I want to make a smaller example to explain what ÷ means. X & ÷ are like brother and sister, or to be more accurate two sides of the same coin. They are inverses (opposites) just like + & - are.

It is easy to see this if we look at a small example like 4 x 3 = 12.

1x	1	2	3	so 1 x 3 = 3
2x	4	5	6	so 2 x 3 = 6
3x	7	8	9	so 3 x 3 = 9
4x	10	11	12	so 4 x 3 = 12

From this we can see clearly that 4 x 3 = 12

We can also see that if we split 12 up into 4 groups, in other words 12 ÷ 4 or $\frac{12}{3}$, we get 3 in each group. So:
12 ÷ 4 = 3
or $\frac{12}{4} = 3$
or 12 shared in 4 equal groups is 3

1x	1	2	3	4	so 1 x 4 = 4
2x	4	6	7	8	so 2 x 4 = 8
3x	9	10	11	12	so 3 x 4 = 12

Here it is clear that 3 x 4 = 12
And similarly if we share our 12 out into 3 equal groups there'll be 4 in each group.

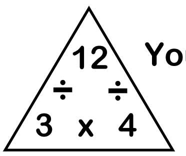
So 12 ÷ 3 = 4
or $\frac{12}{3} = 4$

Just as +5 and -5 being opposites is very useful and important, so too is x4 and ÷4 being inverses incredibly important.

By arranging 12 objects in a rectangle, we can clearly see the link between these 4 truths.

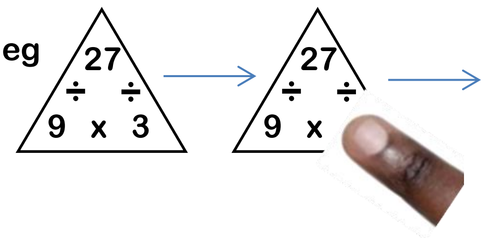
3 x 4 = 12
4 x 3 = 12
12 ÷ 4 = 3
12 ÷ 3 = 4

It is useful to summarise this in a x ÷ triangle



You just cover up the answer!

eg



27 ÷ 9 = 3

There are many memory tricks from poems such as "I ate and I ate and was sick on the floor, 8, 8s are 64." There is the classic 9 times table finger trick, and the lesser known, 6s, 7s, 8s & 9s finger trick. But memorisation comes best, and is most effective for use in maths, when built up through an understanding of where a times table comes from, and how they fit with ÷, using loads of different methods over time. If you just learn them by saying them over and over again you are more like a photo than a mathematician! It is best to have a feel for them, so if I say 63, you almost sense it as part of the 7 and 9 times tables.

Language!

3 x 4 = 12

3 is a factor of 12
4 is a factor of 12
3 and 4 are a factor pair of 12

12 is a multiple of 3
12 is a multiple of 4
12 is the product of 3 & 4

Step 8) 10s, 100s & 1,000s...

Let's try and understand the decimal system a bit better by looking at what happens when you keep adding 10s (or 100s or 1,000s)

We'll look again at the 10 times table.

1x	2x	3x	4x	5x	6x	7x	8x	9x	10x
10	20	30	40	50	60	70	80	90	100

We can see that this could continue counting up in 10s in the same way past 10 lots of 10...

10	20	30	40	50	60	70	80	90	100
110	120	130	140	150	160	170	180	190	200
210	220	230	240	250	260	270	280	290	300
310	320	330	340	350	360	370	380	390	400
410	420	430	440	450	460	470	480	490	500
510	520	530	540	550	560	570	580	590	600
610	620	630	640	650	660	670	680	690	700
710	720	730	740	750	760	770	780	790	800
810	820	830	840	850	860	870	880	890	900
910	920	930	940	950	960	970	980	990	1,000

You can see that this is just like the grid from stage 2, counting from 1 to 100 (+1 each time) but each number has an extra zero. When we multiply any whole number by 10, we can just add a zero.

Similarly you can see that the far right hand column is counting in hundreds,
100, 200, 300, 400, 500, 600, 700, 800, 900, 1,000 and it's like counting in units but with two extra zeros.

The same is true for thousands but with 3 zeros!
1,000; 2,000; 3,000; 4,000; 5,000; 6,000...

This is the wonder of the decimal system.

By definition, each column to the left is ten times bigger than the last.

Ten Thousands	Thousands	Hundreds	Tens	Units
10,000	1,000	100	10	1

1,000x10 100x10 10x10 1x10

So let's take 526x10

Ten Thousands	Thousands	Hundreds	Tens	Units
		5	2	6

500x10 =5,000 20x10 =200 6x10 = 60

So we get

Ten Thousands	Thousands	Hundreds	Tens	Units
	5	2	6	?

500x10 =5,000 20x10 =200 6x10 = 60

We have to add a 0 in the units column, otherwise when we write it down it'll still say 526, which is what we started with before x10.

So 526 x 10 = 5,260

If we times by 100, that is timesing by 10 twice, so we add 2 zeros.

34 x 100 = 3,400

With 1,000 we add 3 zeros.

84 x 1,000 = 84,000

For ÷, as it is the inverse (opposite) of x, we simply remove zeros.

5,260 ÷ 10 = 526 (remove 1 zero)
3,400 ÷ 100 = 34 (remove 2 zeros)
84,000 ÷ 1,000 = 84 (remove 3 zeros).

To find out how to remove zeros, when there aren't enough zeros, you'll need to look at the decimals super topic.

Let's look at a huge number!
23,456,765

This is 23 Million, 456 Thousand, 7 hundred and sixty five.

If we do 23,456,765 x 10
We get 234,567,650 which is
234 million, 567 thousand, 650.

Step 9) Rounding

To round a number means to reduce the number of non-zero digits to make it easier to sense or feel the size of the number.

For example it is easier to sense or feel the size of 200, than of 217, so we could say that 200 is a rounder number than 217.

It is also easier to sense or feel the number 220 than the number 217.

200 is actually 217 rounded to the nearest 100.
220 is 217 rounded to the nearest 10.

Let's look at rounding to the nearest 10.

We'll round the number 37 to the nearest 10.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

If we count up from 37, the first multiple of 10 we get to is 40.

When we count down from 37 the first multiple of 10 we get to is 30.

So the nearest tens above & below 37, are 40 & 30.

But which one is the nearest 10. Here's a nice way writing this, invented by Ethical Tutors student Anushka, putting the nearest 10 above the number, physically above it, and the nearest 10 below the number physically below it.

Just a note that we use nr as a standard shortening for nearest when rounding.

nr 10 above	40	+ 3
Number	37	
nr 10 below	30	- 7

We can clearly see that 40 is nearer (it is 3 away, where 30 is 7 away).

So 37 = 40 (nr 10)

It is very important that we write nr 10 in brackets. This tells us that the nearest 10 to 37 is 40, in other words when we round 37 to the nr 10 we get 40. If we don't write the nr 10 in brackets we are saying that 37 = 40, which is total nonsense. 37 is not equal to 40 (written 37 $\neq$ 40). But 37 = 40 (nr 10)s

It is a lot of work to write out the words nr 10 above/below part each time. It is easier just to write the numbers, the distances, and circle the nearest one then write the answer below.

(40) + 3
37
30 - 7

So 37 = 40 (nr 10)

Let's try 32

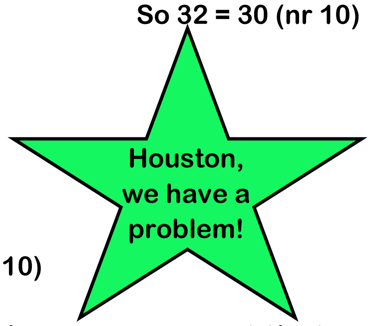
40 + 8
32
(30) - 2

So 32 = 30 (nr 10)

And how about 35?

40 + 5
35
30 - 5

So 32 = ?? (nr 10)



Mathematicians made an agreement that we always round up when we are an equal distance from both 10s (or 100s or 1,000s). This is always when there's a 5 at the end, so we can also say... "Always round upward when there's a 5 at the end."

(40) + 5
35
30 - 5

So 32 = 40 (nr 10)

In general we can say that:

Round Upward
when 5, 6, 7, 8 or 9 at the end

Round Downward
when 0, 1, 2, 3 or 4 at the end

The reason that 5s round upward, makes sense when we look at 100s, which we will do now. We'll look at one that rounds upward & one that rounds downward.

300 + 61
239
(200) - 39

So 239 = 200 (nr 100)

&

(300) + 29
271
200 - 71

So 271 = 300 (nr 100)

Now let's look at the 100s middle point of 50.

(300) + 50
250
200 - 50

So 250 = 300 (nr 100)

It isn't very satisfying to round up on 5s, just because MOLE told you to. Let's look at 249 & 251 to see why

300 + 51
249
(200) - 49

So 239 = 200 (nr 100)

(300) + 49
251
200 - 50

So 250 = 300 (nr 100)

We can see from this that anything from 251 up to 299 will round up to 300. If we make the agreement (as mathematicians have) that a 5 rounds up (here representing 50 in the tens column), then we don't even have to look at the units. If the tens is a 5, then it must either be 250, 251, 252, 253, 254, 255, 256, 257, 258 or 259, so if we agree that 250 (the midpoint) rounds up to 300, then all these with a 5 in the tens round up to 300, just as all those with a 4 in the tens (240, 241, 242, 243, 244, 245, 246, 247, 248 & 249) round down. This way we don't have to look past the 10s when rounding to nr 100, or the 100s when rounding to nr 1,000.

We can write this much neater way, with a line separating the digit that may change, and the digit telling us whether it changes or not.

(7 | 0 0)
6 | 5 7 — 5, so round upward
6 | 0 0

So 657 = 700 (nr 100)

&

2 | 0 0
1 | 2 4 — 2, so round downward
(1 | 0 0)

So 124 = 100 (nr 100)

We can round to nr 10 or 1,000 in the same way

(9 | 0)
8 | 9 — 9, so round upward
8 | 0

So 89 = 90 (nr 10)

&

5, | 0 0 0
4, | 4 3 3 — 4, so round downward
(4, | 0 0 0)

So 4,433 = 4,000 (nr 1,000)

Step 10) + Integers.

Let's look at 345 + 214
We'll break it up into parts.

Hundreds	Tens	Units
300	40	5
200	10	4
500	50	9
300 + 200	40 + 10	5 + 4

So it is 500 + 50 + 9 = 559
345 + 214 = 559

But what if the number from adding the units is too big for the unit's column?

Let's try 473 + 198

Hundreds	Tens	Units
400	70	3
100	90	8
600	70	1
100	10	

70 + 90 + 10 =170 = 100 + 70

3 + 8 = 11 = 10 + 1

So 473 + 198 = 600 + 70 + 1 = 671

You might have noticed that everything in the tens column had 1 zero, and everything in the hundreds column had 2 zeros, so we can just write the whole thing without all the zeros and the column tells you whether it's a 10, 100 or 1,000.

3	4	5	
2	1	4	+
5	5	9	

&

4	7	3	
1	9	8	+
6	7	1	
1	1		

Step 11) – Integers

Let's do 874 - 251

Hundreds	Tens	Units
800	70	4
200	50	1
600	20	3
800 – 200	20 – 50	4 – 1

Which is 600 + 20 + 3 = 623
So 874 – 251 = 623

But what happens if the number you have to take away (in the second row of the units/10s/100s column) is bigger than the number you started with (in the first row of the units/10s/100s column)?

Let's use 725 – 347

10 – 40? Borrow 10 from the 10s

5 – 7? Borrow 10 from the 10s

Hundreds	Tens	Units
600 ~~700~~	110 ~~20~~	¹5
300	40	7
300	70	8
500 – 300	110 – 40	15 – 8

Leave 300 + 70 + 8 = 378
So 725 – 347 = 378

Just like with +, we can write these out in quicker (but less understandable) form, by missing off the zero in the 10s, both zeros in the 100s, and so on. Like so:

8	7	4	
2	5	1	–
6	2	3	

&

6 ~~7~~	11 ~~2~~	15	
3	4	7	–
3	7	8	

Step 12) x digit by an integer

We'll play with 5 x 14 or 5(10 + 4)
(which makes 70)

We'll make 5 rows of 14 rectangles, splitting each row into 10 and 4.

← 14 rectangles (each row) →

1	2	3	4	5	6	7	8	9	10	1	2	3	4
2										2			
3										3			
4										4			
5										5			

Left: 5 rows, with 10 in each row, 5 x 10 = 50

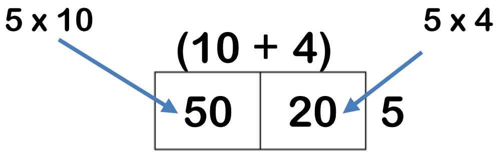
Right: 5 rows, with 4 in each row, 5 x 4 = 20

The total number of rectangles is 50 + 20 = 70

So 5 x 14
= 5(10 + 4)
= 5x10 + 5x4
= 50 + 20
=70

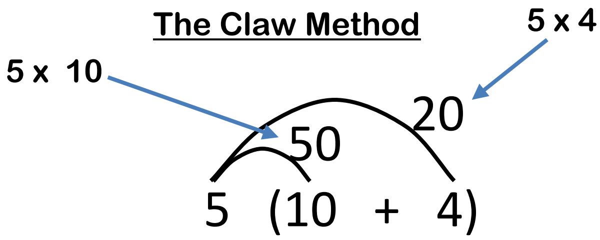
There's a few different good ways to do this.

<u>Grid Method</u>



50 + 20 = 70

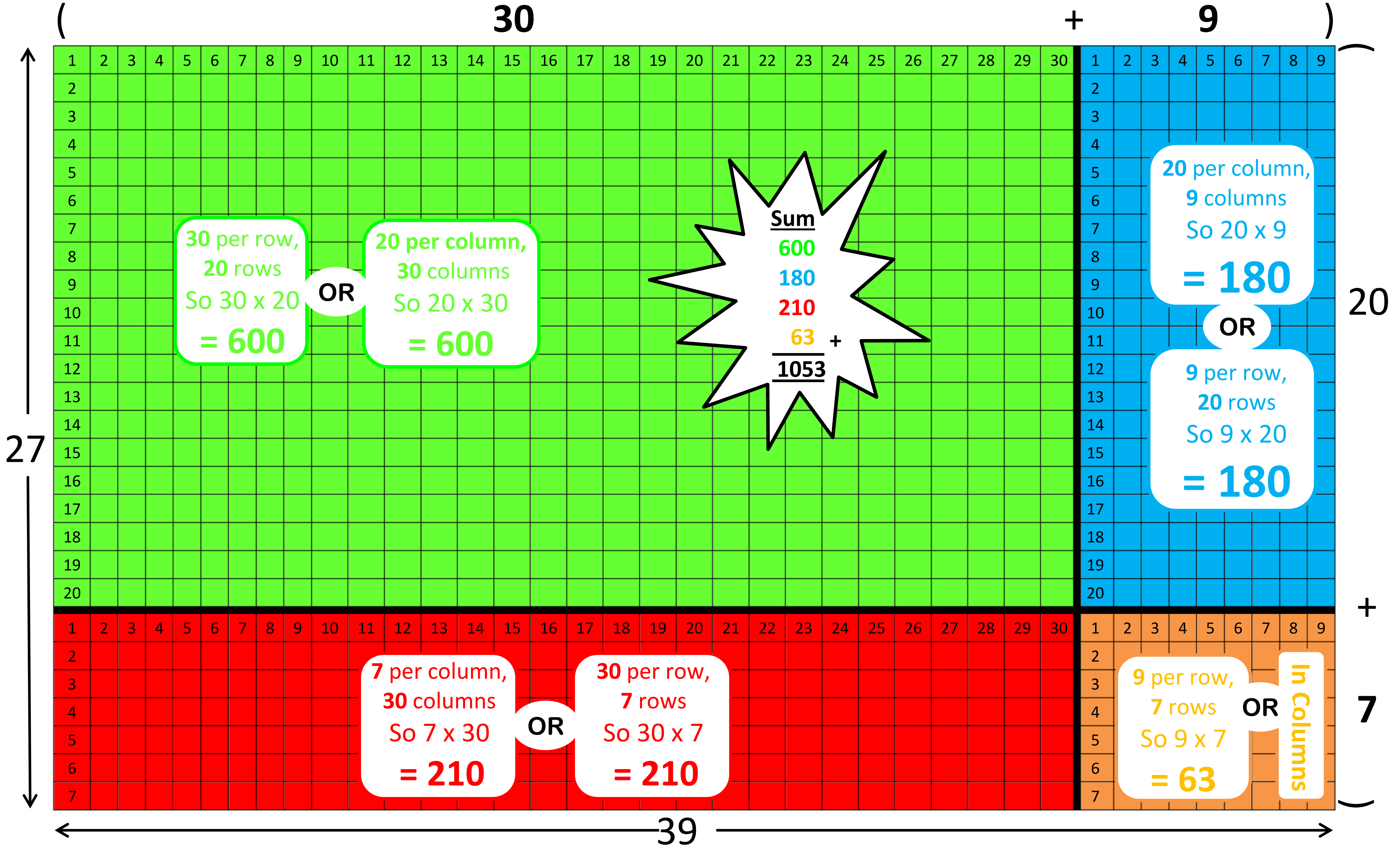
<u>The Claw Method</u>



Step 13) Integers x Integers

1) The Grid of Meaning

Let's work with 39 x 27 = 1053, which can be written (30 + 9)(20 + 7). The grid of meaning shows us why all other methods work. We lay out our 39 x 27 rectangles in a grid, with 27 rows with 39 rectangles in each row. We then divide this grid in to four parts, cutting the rows into 30 + 9, and the columns into 20 + 7. We find (using rows & columns) the number of rectangles in each part, and add them together.



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The grid of meaning is the most concrete method. All other methods derive from the grid of meaning. They are all other ways of getting the 4 parts (600, 180, 210 & 63 in our eg)

Grid Methods
1) The Grid of Meaning (on last page)
2) The Sketch-Grid
3) The Slow Snail (With 0s)
4) The Speedy Snail (No 0s)

Horizontal Methods
5) Claw
6) Smiley Face
7) FOIL

Vertical Methods
8) Bow Tie Method
9) Column Method

Technological Methods
10) Calculator
11) Spreadsheet

1) The Sketch-Grid Method.

Just like the grid off meaning, but a sketch of it, you don't actually draw 39 by 27 rectangles.

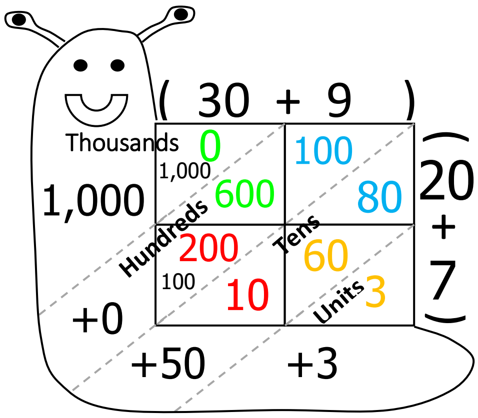
(30 + 9)

30	9	
600	180	20 +
210	63	7

Sum = 1053

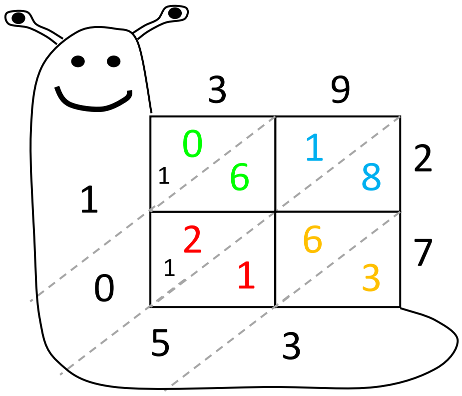
The snail uses diagonal lines on a grid, to include the adding in the grid.

3) The Slow Snail Method (with Zeros)



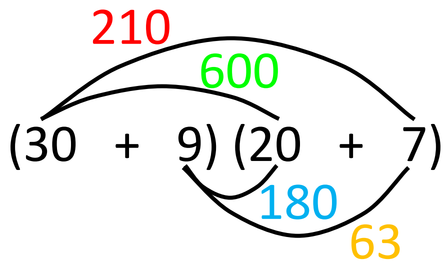
With the slow snail method, the ten's diagonal all have 1 zero, the 100s 2 zeros and the 1,000s 3 zeros. From this we can make a speedy snail method, where we don't write all these repeated zeros.

4) The Speedy Snail Method (no zeros)

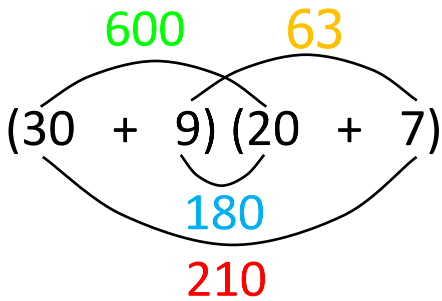


The horizontal methods (Smiley Face, Claw & FOIL) allow us to get the 4 parts directly from the brackets, which here are (30 + 9)(20 + 7)

5) The Claw Method



6) The Smiley Face Method



The smiley face method has been used in schools for years, but it doesn't look that much like a smiley face. Ethical Tutors student Nina invented a better version called the claw.

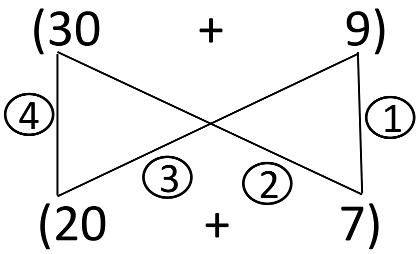
7) The FOIL Method.

39 x 27
= (30 + 9) (20 + 7)

First = 30 x 20 = 600
Outside = 30 x 7 = 210
Inside = 9 x 20 = 180
Last = 9 x 7 = 63

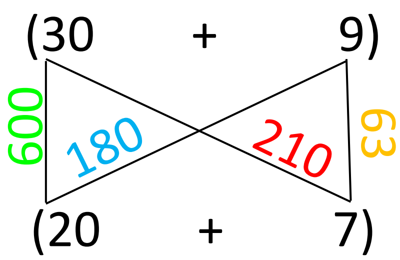
8) The Bow Tie Method

This was invented by Ethical Tutors student Anushka. Originally called the Flag Method, it was renamed the Bow Tie Method by Ethical Tutors student Emily. It provides a pictoral way to understand the column method (shown after).



① 7x9
② 7x30
③ 20x9
④ 20x30

Above shows you the order and calculations, below what you might actually write.



63
210
180
600
1053

9) The Column Method

Surely the fastest method, but also the most abstract, and hardest to make sense of – but comparing to the Bow Tie above helps!

39
x 27

① 7x9 = 63
② 7x30 = 210
③ 20x9 = 180
④ 20x30 = 600

1053

Here we've given a line for all 4 parts

In the full, fastest and (honestly) final version, the 7x39 is written on the first line, and the 20x 39 on the second line

39
x 27

7x30 and 7x9 → 273 (carry 6)
20x30 and 20x9 → 780 (carry 1)

1053

A nice investigation here, is to look at multiplying 3-digit (or bigger) numbers. You can go through all the methods and see which ones you can extend to more digits, and which ones only work for 2-digit x 2-digit numbers. Enjoy…

Step 14) integer ÷ digit

Inverses, or opposite functions can be thought of as questions.

So 10-3 can be thought of as the question, what do I need to add to 3 to make 10.

Similarly, 36÷9 can be thought of as the question how many times 9 makes 36. Well 4x9 makes 36, so 36÷9 = 4.

We can do the same with a larger question like say 693÷3.

We can build up in multiples of 3 till we get to 693. This is sometimes called chunking as you build up in chunks of multiples of 3.

Chunk	Gives	Left
		693
100 x 3	300	393
100 x 3	300	93
10x3	30	63
10x3	30	33
10x3	30	3
1x3	3	0

In this chunking we took of multiples of 3 till we got to 0. We took of 100, then another 100, then 10, then 10, then another 10, then finally just one last 3. In total we took off…

100 + 100 + 10 + 10 + 10 + 1 = 231

So 231 x 3 = 693
and hence 693÷3=231

In this one we chunked down, by taking multiples of 3 away from 693 until we got to 0.

We can also chunk up, starting from zero, until we make the desired number. Let's look at chunking up with 1,141

This is bigger than 700 so we can start with a 100x7 chunk.

Chunk	Gives	Total
		0
100 x 7	700	700
50x7	350	1050
10x7	70	1120
3x7	21	1141

This can be easier in that it is building the number of 7s you want from the ground up. But it can be harder to see what size chunk to go for next. If you are chunking down, you can only use a 100x7 multiple if you have 700 left. But if you are chunking up, say when we got to 700, we only had 441 left, but would have to calculate 1141 – 700 to know this.

With addition, subtraction and multiplication, we had fast methods all based on splitting the numbers into hundreds, tens and units, and there is a similar method for division.

This method us known as the bus stop method. Let's look at 693÷3. You write the starting number under the bus stop and the divisor outside the bus stop…

3	600	90	3
	Hund.	Tens	Units

Then you start with the largest number and divide it.

$\frac{600}{3}$ → 200, $\frac{90}{3}$ → 30, $\frac{3}{3}$ → 1

	200	30	1
3	600	90	3
	Hund.	Tens	Units

So $\frac{693}{3} = 231$

And of course this can be done more quickly without writing 2 zeros in the hundreds column, or 1 zero in the 10s.

$\frac{6}{3}$ → 2, $\frac{9}{3}$ → 3, $\frac{3}{3}$ → 1

	2	3	1
3	6	9	3
	Hund.	Tens	Units

So $\frac{693}{3} = 231$

However, the answers when dividing hundreds aren't always nice multiples of 100…

Let's look at 978÷3

$\frac{900}{3}$ → 300, $\frac{70}{3}$ → 20, $\frac{18}{3}$ → 6

	300	20	6
3	900	70	$^{1}8$
	Hund.	Tens	Units

The 70÷3 gives us 20, and 10 left still to be divided, which we add into the units column. Of course 70÷3 could also be written as 23, with only 1 to carry, but we can't write 23 in the tens column, as it is not a multiple of ten, so we write 20, and carry 10 to be divided with the 8 in the units column.

This can all be written in shorthand…

$\frac{9}{3}$ → 3, $\frac{7}{3}$ → 2, $\frac{18}{3}$ → 6

	3	2	6
3	9	7	$^{1}8$
	Hund.	Tens	Units

Here the carrying makes more sense, as 7÷3 = 2 (remainder 1 to be carried)

So $\frac{978}{3} = 326$

But what happens if you still have some left at the end?

Let's try 375÷2 (in shorthand)

	1	8	7	remainder 1
2	3	$^{1}7$	$^{1}5$	

But this remainder 1 still needs to be divided by 2, and 1÷2 is ½ so we could write…

$\frac{375}{2} = 187r1$

or $\frac{375}{2} = 187\frac{1}{2}$

We can actually use the bus stop to turn it straight into a decimal. Note that 375 = 375.0

	1	8	7	.	5
2	3	$^{1}7$	$^{1}5$	.	$^{1}0$

so $\frac{375}{2} = 187.5$

Now ½=0.5 so I makes sense that one way you get $187\frac{1}{2}$ and another way you get 187.5 because they are the same thing!

You can even use this method to demonstrate facts like $\frac{3}{4} = 0.75$ and $\frac{1}{3} = 0.\dot{3}$

$$\frac{3}{4} = \frac{3.0000}{4}$$

	0	.	7	5		
4	3	.	$^{3}0$	$^{2}0$	0	0

so $\frac{3}{4} = 0.75$

	0	.	3	3	3	3…
3	1	.	$^{1}0$	$^{1}0$	$^{1}0$	$^{1}0$

You can see that this pattern will just continue so

$$\frac{1}{3} = 0.3\cdots = 0.\dot{3}$$

Step 15) BIDMAS

BIDMAS is a topic all about the order we do maths operations. Let's discuss 3 + 5 x 2

3 + 5 = 8
And then 8 x 2 = 16
So 3 + 5 x 2 = 16

But what happens if we did the 5 x 2 first?
5 x 2 = 10
And then 3 + 10 = 13
So 3 + 5 x 2 = 13

Oh dear, we have two different values for 3 + 5 x 2, one is 13, and the other is 16.

Mathematicians realised this and over time evolved a system that all mathematicians around the world now agree to use, so that whether you are in the North Pole, or in the South of Brazil, 3 + 5 x 2 = 13 and never 16.

This is because we always do operations in the order given by BIDMAS

1) Brackets
2) Indices (Powers)
3) Division
4) Multiplication
5) Addition
6) Subtraction

So in the example we just did, 3 + 5 x 2, we have addition and multiplication. Looking down the BIDMAS list multiplication comes first so…

5 x 2 = 10
And then 3 + 10 = 13
So 3 + 5 x 2 = 13

Notice that brackets come before everything else so if we had (3 + 5) x 2 then we'd have to do the brackets first and we'd then get…

3 + 5 = 8.
And then 8 x 2 = 16
So (3 + 5) x 2 = 16

So 3 + 5 x 2 = 13
and (3 + 5) x 2 = 16

We can use this order for any maths calculation, and any two mathematicians, anywhere in the universe will get the same answer!

Let's try one more $50 – 3 \times 2^2$

First is the power so $2^2 = 4$

$50 – 3 \times 2^2 = 50 – 3 \times 4$

Multiplication before subtraction
s0 50 – 12 = 38

$50 – 3 \times 2^2 = 38$