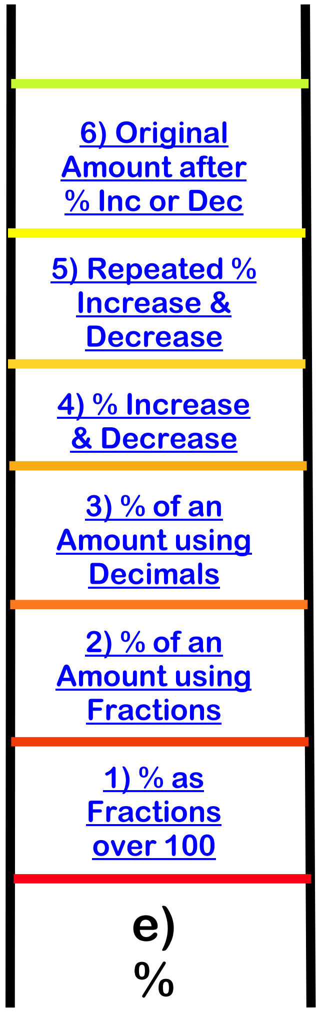
e) Understanding%



Step 1) % as Fractions over 100

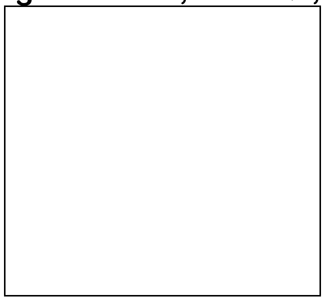
A percentage comes for the word cent, which means 100. Both in US dollars and the European Euro, or even the old French Francs, a cent was a coin, 100 of which are worth one of the main currency coins, be it a $, a € or a FF (French Franc).

The per part means how many out of 100.

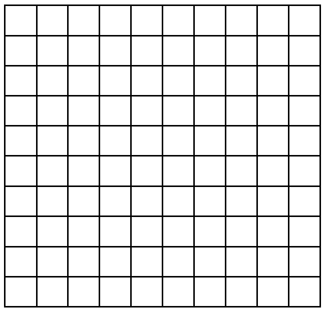
So 1% means 1 out of every 100.

This makes a clear link to fractions where $\frac{1}{100}$ can be read as 1 over 100 (its position)
1 divide by 100 (like the cent coins)
or 1 out of 100 (which links to %s)

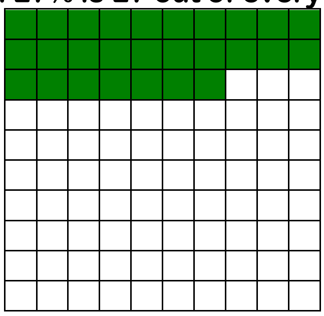
So representing the whole, or 100%, as a square...



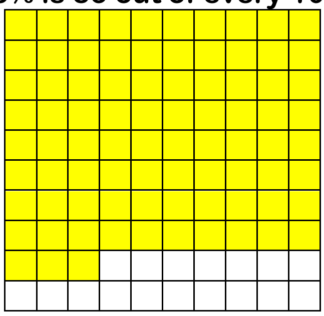
We can cut it into 100 equally sized pieces and each one is 1%.



Then 27% is 27 out of every 100.



83% is 83 out of every 100.



Percentages are a very useful of understanding proportion or how much of the whole there is. We could use a system of perdecs... or out of every 10. This would be easily recognisable to most people, as most people have a sense of the scale 1 to 10 as sizes. Often people use the phrase, on a scale of 1 to 10 how angry are you, how much do you like chocolate, how big is the hill we have to cycle up etc. A number between 1 and 10 would provide a meaningful answer to this question.

But the scale doesn't provide a lot of precision... the difference between 5 out of 10 and 6 out of 10 could be huge for an individual opinion.

So the %, thinking about things as how many out of 100 gives a very accurate idea of how much of some possible total there is, but can still be understood by most people. It is clear that 84% is loads more than 23%, and a little more than 81% (though both 84%

and 81% would be seen as the same... 8 out of 10, on an out of 10 scale) and we still have a sence of all of these as how much of the total 100% they are.

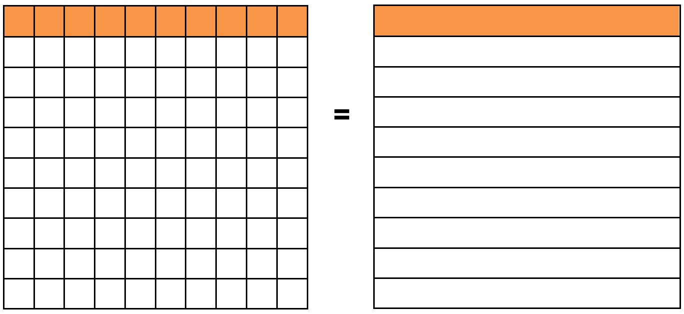
Step 2) % of an Amount using Fractions

To find a percentage of an amount, it is at first easiest to understand them as fractions and use the fractions to find the % of the amount.

So for example let's look at finding 10% of an amount.

10% as a fraction is $\frac{10}{100}$ which simplifies to $\frac{1}{10}$

10% = $\frac{10}{100}$ = $\frac{1}{10}$



So in finding 10% of an amount, we can actually find $\frac{1}{10}$ of that amount, which we do by dividing by 10 (See fractions step 5).

For example
10% of 60
= $\frac{1}{10}$ of 60
= 60 ÷ 10
= 6

With a slightly more complex number let's find
10% of 734
= $\frac{1}{10}$ of 734
= 734 ÷ 10
= 7.34

We can use this to find much more complex percentages of amounts.

For example to find 35% of 60 we can use the above result.

10% of 60
= $\frac{1}{10}$ of 60
= 60 ÷ 10
= 6

Now 30% of 60
= 3 x 10% of 60
= 6 x 3
= 18

And 5% of 60
= (10% of 60) ÷ 2
= 6 ÷ 2
= 3

Finally 35% of 60
= 30% of 60 + 5 % of 60
= 18 + 3
= 21

This method can be used for very complex percentages.

Let's look at 78% of 50

78 % of 50 = 7x10% of 50 + 8 x 1% of 50

10% of 50 = 5
70% of 50
= 7 x 5
= 35

1% of 50 = (10% of 50)÷10
=5÷10
= 0.5

so 8% of 50
= 8x1% of 50
= 8 x 0.5
= 4

Now 78% of 50 = 70% of 50 + 8% of 50
= 35 + 4
= 39

This is quite a nice way to break down a complex % into little bite sized chunks

eg 27% = 10% + 10% + 5% + 1% + 1%

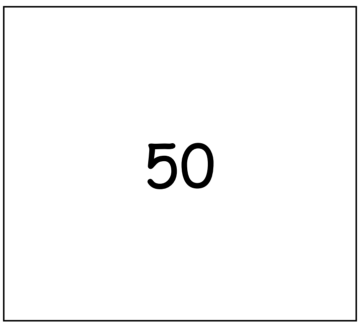
However it takes quite a lot of work!

Another really nice method is the 1% method.

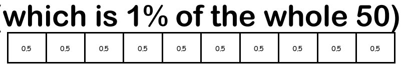
This is very similar to the unitary method from proportionality.

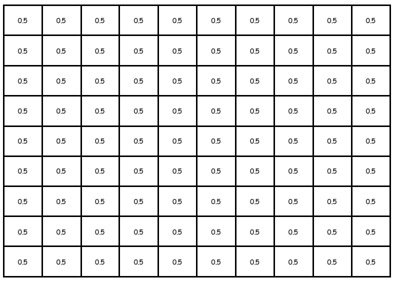
with 78% of 50
= 78 x 1% of 50
Now 1% of 50 is 50 ÷ 100 = 0.5
so 78% of 50 = 78 x 0.5 = 39

In visual terms

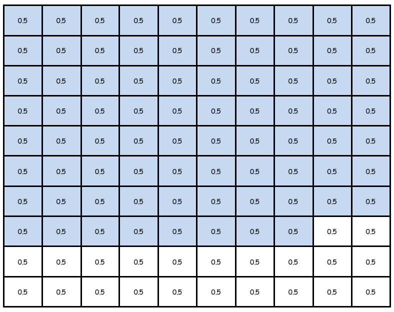


Cutting the 50 into 100 equal bits each one is 0.5 (which is 1% of the whole 50).





And so 78% of 50 is 78 of these 0.5 pieces.



and 78 x 0.5 = 39

Let's look directly at the fraction over 100 without breaking it into parts.

35% of 60

$=\frac{35}{100}\times 60$

=21

Or

78% of 50

$= \frac{78}{100}\times 50$

= 39

All of these methods work, but they all require doing more than one step. Even with a calculator they all require doing at least two things. ÷100 then x78, or x78 then ÷100.

However, if I am going to use a calculator, why not just use the fraction over a hundred converted to a decimal as this would require only one step.

So once you have mastered building up percentages of amounts by breaking them into fractions, the next step is finding percentages of amounts using decimals.

Step 3) % of an Amount using Decimals

From here on in it gets much more abstract, but the calculations become much easier. The understanding needs to be built in well on steps 1 & 2 before even considering step 3.

Though everything from step 3 onwards can be done with a written method, they would usually be done with a calculator.

Let's look at

35% of 60

= 0.35 x 60

=21

Or

78% of 50

= 0.78

= 39

This is very similar to using the fractions as $\frac{35}{100} = 0.35$ and $\frac{78}{100} = 0.78$

but with the decimal you only have one thing to type in (rather than doing a multiply, and then dividing by 100 which is 2 steps) and this will be a big advantage later when we come to do more complex things with percentages.

Step 4) % Increase & Decrease

To increase or decrease an amount by a % means to take that % of the amount and then add it on to the original amount (increase) or take it away from the original amount (decrease).

From this step onwards we are always going to use the decimal method to work out percentages of amounts, but it is essential that you have understood the previous steps well to understand how they all fit together. Using different fractions to build up the percentage, using the fraction of 100, or using the 1% method are all equivalent (but more complicated than) using the decimal method.

So let's look at increasing 60 by 15%

15% of 60

= 0.15 x 60

= 9

Adding this 15% of 60 (which is 9) on to the original 100% of 60 (which is 60) gives us the increased total.

60 + 9 = 69

So when I increase 60 by 15% I get 69

HOWEVER

We are back to using two steps...

Can we do it in 1 step.

At the end I added my 100% to my 15% increase... so overall I am finding 115%

115% of 60

= 1.15 x 60

= 69

Ta da! It gives the same answer. I call this the single multiplier method. It is always a simpler calculation if you can understand how to multiply your starting amount by a single multiplier (in this case 115% also known as 1.15) rather than finding a percentage and then doing an extra step adding it on.

Now we'll decrease 60 by 15%

15% of 60
= 0.15 x 60
= 9

The original 100% – the 15% decrease
= 60 – 9
= 51

But using a single multipler we can use
100% - 15%
= 85%

85% of 60
= 0.85 x 60
= 51

So with % increase or decrease all we need to know is the single multiplier.

So...

To increase by 17% we times by 1.17
To increase by 53% we times by 1.53
To increase by 75% we times by 1.75
To increase by 110% we times by 2.1
and so on...

And...

To decrease by 10% we times by 0.9
To decrease by 30% we times by 0.7
To decrease by 17% we times by 0.83
To decrease by 82% we times by 0.18

5) Repeated % Increase & Decrease

A repeated % increase or decrease, is where we continually add or take a way a given percentage of an amount.

The most frequent example of this is with profit or loss of the value of things. When we make a profit on the value of something we sometimes call it interest (particularly when this is a fixed percentage given to us by a bank out of the – usually larger – amount that they make from our investment).

The basic type of interest is called simple interest. This is where the interest is only paid on the original amount.

Let's look at a 10% per year simple interest on an investment of £60 over 3 years.

We find 10% of 60 = 6
And there are three years of this same fixed amount, so we add 3 x 6 = £18

£60 + £18 interest gives £78

Now if I was investing this money, I would say "Hang on a minute, after the first year, I had £6 interest so in the second year I was really investing £66 not just £60."

And of course I would be totally right. This is called compound interest. Compound shapes are where more than one shape is put together, compound interest is where the new "part" added after the first years interest, is also part of the investment for the second year (and so on).

Let's look at compound interest (it is almost always the one we actually use except in old fashioned maths exams).

With compound interest at 5%, how much will you have after 4 years with an initial £345 investment?

The original amount is 100%
100% + 5% = 105%
For 105% we use a multiplier x1.05

Year	Start	Multiplier	End
1	345	x 1.05	362.25
2	362.25	x 1.05	380.36
3	380.36	x 1.05	399.38
4	399.38	x 1.05	419.34

So after 4 years we have £419.35

It is interesting (no pun intended) to compare this to the total after simple interest is added...
4 (years) x 0.05 (% increase) x 345 (investment)
= £69 added
Add it on to the investment is 345 + 69 = 414 which is over £5 less than with compound interest.

This is because of all the extra little bits of interest paid on last years interest. Over many years compound interest can leave you with hundreds of times more cash than simple interest.

The problem with this method is that we are back to lots of steps. Every year you have to x1.05. But overall what we have done is...
375 x 1.05 x 1.05 x 1.05 x 1.05
which is the same as
375×1.05^4

This is the single multiplier method for repeated % increase.

Let's look at a repeated % decrease example...

A second hand car costs £5,000 but loses 15% of it's value every year. What is it worth after 3 years?

Original amount is 100%
100% - 15% = 85%

For 85% of an amount x 0.85

Year	Start	Multiplier	End
1	5,000	x 0.85	4250
2	4250	x 0.85	3612.5
3	3612.5	x 0.85	3070.63

= £3070.63

But with the single multiplier method, we can simply times the original amount by 0.85^3

5,000 x 0.85^3
= £3070.63
Much easier!

So...

To increase by 7% for 5 years we times by 1.07^5
To increase by 25 % for 7 years we times by 1.25^7
To increase by 10% for 2 years x 1.1^2
and so on...

And...

To decrease by 10% for 4 years we times by 0.9^4
To decrease by 40% for 9 years we times by 0.6^9
To decrease by 23% for 3 years x 0.67^3

This single multiplier method becomes essential for working backwards in the next step.

6) Original Amount after % Increase or Decrease

After a 17% increase I have 99.45. What was the original amount?

To answer these what was an original amount questions, we have to work out what we did to the original amount and reverse (or invers) it.

So if the original amount was increase by 17% then we had to multiply the original amount x 1.17

So if we do the opposite (or inverse) of x 1.17 to the final amount, we'll get back to the orginial amount.
The inverse of x 1.17 is ÷ 1.17

$$\frac{99.45}{1.17} = 85$$

So the original amount was 85

We work in the same way with the original amount after a % decrease. After a 9% reduction I have 141.96. What was the original amount?
9% decrease is original amount x 0.91
so original amount is

$$\frac{141.96}{0.91} = 156\frac{141.96}{0.91} = 156$$

so the original amount was 156

We can even quickly and easily work out the original amount after several years of repeated % increase simply by taking the inverse of what you did to find the repeated % increase.

After 3 years at 7% interest I have 1225.043.
What was the original amount?
7% increase for 3 years is original amount x 1.07^3
The opposite of x 1.07^3 is ÷ 1.07^3
so original amount is

$$\frac{1{,}225.043}{1.07^3} = 1{,}000$$