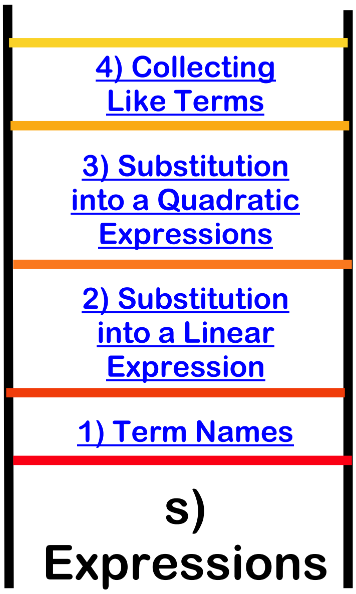
s) Expressions.



Step 1) Term Names

Variable numbers are numbers that change depending on the situation. Income changes depending on how many hours you work, and changes each time you get a pay rise. The depth of a hole changes as long as you keep digging it. The number of people in your learning room changes each day, sometimes it goes up, sometimes it goes down.

To allow us to play with variable numbers (we sometimes calling playing with variable numbers algebra), and in particular so we can write them down, we use letters.

An expression is a group of terms, that use variable numbers (usually represented as letters) to describe (or express) a situation in real life.

There are different types of terms:

Constant terms: 4, 9, -23, 7½, -3.2 are examples. These terms are constant numbers, numbers which don't change, which you studied lots in the area of maths called numbers (or constant nos).

Linear terms: These are terms with an x that is not squared and not cubed: $x,\ 4x,\ 9x,\ -7x,\ 2\frac{1}{2}x$ We often use x, y & z as they are right at the end of the alphabet and not so often used in regular words, but any letter can be a variable: a, b, c, d, e, f ...

Quadratic terms: These are terms with an x^2 said x squared: $x^2,\ 5x^2,\ 7x^2,\ -3x^2,\ \frac{1}{4}x^2$. Easy to remember as a square is a regular **quad**rilateral (4-sided polygon/shape).

Cubic terms: These are terms with an x^3 said x cubed: $x^3,\ 4x^3,\ 19x^3,\ -x^3,\ \frac{2}{3}x^3$.

These types of terms (constant, linear, quadratic and cubic) must always be kept separate and can't be mixed up as we'll see in step 4 when we start trying to collect them.

The power of x has no limit. You can have x^4 (called a quartic term), x^5 (called a quintic term) and so on... But knowing term names up to x^3 (cubic terms) is more than enough for most mathematicians! The terms can just keep on getting bigger (or more POWERful) $x^5,\ x^6, x^7$ ...

Step 2) Substitution into a Linear Expression.

A linear expression is one who's largest term is a linear or x term. It can also have smaller terms, which here would be a constant term.

So $x+5,\ \ 4x-3,\ \ 3x,\ \ 12-2x$ are all linear expressions with two different terms, except $3x$ which is also a linear expression but only has 1 term.

An expression is used to represent (or express) a particular situation.

So for example with $x+5$..

Let's say a squirrel called Ranjit was collecting acorns. If Ranjit already had 5 acorns, and went out foraging and collected "some" more acorns, the total she would have would be "however many she collected" plus the 5 she already had.

But, "how many I happen to have," or "the unknown number," or "the variable number," is a bit of a mouthful. So we shorten this part to x. Now let's try again, it is much easier to read.

Take 2...Let's say a squirrel called Ranjit was collecting acorns. If Ranjit already had 5 acorns, and went out foraging and collected x more acorns, the total she would have would be x plus the 5 she already had.

With this simpler language, we can now just express the number of acorns after the forage as $x+5$.

Similarly for $4x-3$ the situation could be that 4 siblings share their monthly pocket money. They all get the same amount x £s per month. However they also share a monthly subscription to their favourite magazine "Maths Monthly," and this costs them £3 per month. There's 4 of them with x £s each, that's $4x$, but we have to minus the £3 magazine cost. So the total amount they get, after their parents take off the subscription money is $4x-5$.

It is important to remember that variable numbers, though we often learn about them out of context of a real situation, are only useful to mathematicians because they are used to EXPRESS a real situation. We'll look a little more at this when adding like terms (step 4) but after that we will not always come back to a real situation, unless we are problem solving.

So with that in mind, let us take the situation of the shared pocket money. The four siblings shared pocket money, after the magazine subscription was given by $4x-3$

Now at a particular time we might find out how much money they each got per month. Then to work out the total they share, we could just substitute that amount into our expression.

Now we face one of the hardest but easiest things about learning algebra; the reality is that we wouldn't use an algebraic expression for this problem. So the mind says, why use algebra, I can just solve the problem by working out how much money they have. But with a much more complex expression, using algebra would make the calculations muuuuuch easier. So we have to learn algebra with simple expressions that we probably wouldn't actually use algebra for, and then later it becomes veeeery useful for complex things.

So x is the amount each sibling gets in pocket money per month and $4x-3$ is the total money they share after the magazine subscription has been taken off.

Remember $4x$ means $4\times x$.

		$4x-3$
When $x=1$	Calculations	$4\times1-3$
	Term Values	$=4-3$
	Expression Value	$=3$
When $x=2$	Calculations	$4\times2-3$
	Term Values	$=8-3$
	Expression Value	$=5$
When $x=5$	Calculations	$4\times5-3$
	Term Values	$=20-3$
	Expression Value	$=17$

And we can do this for any other expression, we don't really need to know the situation it expresses, we can still substitute in some values for the variable number (usually x) to find out the value of the expression. Let's try a few values of x with the expression $2x+7$

		$2x+7$
When $x=2$	Calculations	$2\times2+7$
	Term Values	$=4+7$
	Expression Value	$=11$
When $x=6$	Calculations	$2\times6+7$
	Term Values	$=12+7$
	Expression Value	$=19$
When $x=9$	Calculations	$2\times9+7$
	Term Values	$=18+7$
	Expression Value	$=25$

The calculation line is useful for learning about substitution, but once we've got the hang of it, writing just the term value line, and then the expression value line is more than enough.

And of course, you wouldn't actually write the words 'Term Values,' and 'Expression Value,' each time. So the substitution table of a skilled mathematician might look like this:

	$2x+7$	$3x-1$	$10x+2$
When $x=2$	$=4+7$ $=11$	$=8-1$ $=7$	$=20+2$ $=22$
When $x=6$	$=12+7$ $=19$	$=18-1$ $=17$	$=60+2$ $=62$
When $x=9$	$=18+7$ $=25$	$=27-1$ $=26$	$=90+2$ $=92$

Step 3) Substitution into a Quadratic Expression.

We will jump straight to the table here, and for learning we'll put the calculation line back in.

		x^2+2x-1
When $x=1$	Calculations	$1^2+2\times1-1$
	Term Values	$=1+2-1$
	Expression Value	$=2$
When $x=2$	Calculations	$2^2+2\times2-1$
	Term Values	$=4+4-1$
	Expression Value	$=7$
When $x=5$	Calculations	$5^2+2\times5-1$
	Term Values	$=25+10-1$
	Expression Value	$=34$

As with the linear expressions, writing the term calculations each time is a bit over the top. Here's a compromise that also kinda shows you why they're not needed.

		$x^2 + 2x - 1$	**Notes**
When $x = 1$	Term Values Expression Value	$= 1 + 2 - 1$ $= 2$	Quad Term: 1^2 Linear Term: 2×1
When $x = 2$	Term Values Expression Value	$= 4 + 4 - 1$ $= 7$	Quad Term: 2^2 Linear Term: 2×2
When $x = 5$	Term Values Expression Value	$= 25 + 10 - 1$ $= 34$	Quad Term: 5^2 Linear Term: 2×5

Most mathematians don't write the notes or a calculation line – they'd just write…

	$x^2 + 2x - 1$
When $x = 1$	$= 1 + 2 - 1$ $= 2$
When $x = 2$	$= 4 + 4 - 1$ $= 7$
When $x = 5$	$= 25 + 10 - 1$ $= 34$

As you will discover in all areas of working with quadratics, it is easier when you have only one x^2 , and harder to work with $2x^2, 3x^2, 4x^2$ … & so on.

Let's try few harder ones…

	x^2	$2x^2$	$3x^2$	$10x^2$
When $x = 1$	1^2 $= 1$	2×1^2 $= 2 \times 1$ $= 2$	3×1^2 $= 3 \times 1$ $= 3$	10×1^2 $= 10 \times 1$ $= 10$
When $x = 2$	2^2 $= 4$	2×2^2 $= 2 \times 4$ $= 8$	3×2^2 $= 3 \times 4$ $= 12$	10×2^2 $= 10 \times 4$ $= 40$
When $x = 5$	5^2 $= 25$	2×5^2 $= 2 \times 25$ $= 50$	3×5^2 $= 3 \times 25$ $= 75$	10×5^2 $= 10 \times 25$ $= 250$

Because of BIDMAS the squaring always happens before multiplying by the constant (here 2, 3 and 10).

Let's see a good quadratic substitution table (with one cubic thrown in for fun!).

	$x^2 + 2x - 1$	$3x^2 - 4x + 3$	$x^3 + 5$
When $x = 1$	$1 + 2 - 1$ $= 2$	$3 - 4 + 3$ $= 2$	$1 + 5$ $= 6$
When $x = 2$	$4 + 4 - 1$ $= 7$	$12 - 8 + 3$ $= 7$	$8 + 5$ $= 13$
When $x = 5$	$25 + 10 - 1$ $= 34$	$75 - 20 + 3$ $= 58$	$125 + 5$ $= 130$

Step 4) Collecting Like Terms.

Why can we add like terms?

Let's look at $2x + 3x \equiv 5x$

The $\equiv$ symbol doesn't mean equals, it means they are always the same in any situation. So you could swap an $(2x + 3x)$ for a $(5x)$, and vica versa at any time in any algebraic expression!!!

On a basic level, we can substitute in a few different numbers and be convinced that 2 lots of a number ($2x$), and 3 lots of a number ($3x$), together will always make 5 lots of that number ($5x$).

	$2x$	$3x$	$2x + 3x$	$5x$
When $x = 1$	**2**	**3**	**2 + 3 = 5**	**5**
When $x = 3$	**6**	**9**	**6 + 9 = 15**	**15**
When $x = 5$	**10**	**15**	**10 + 15 = 25**	**25**
When $x = 10$	**20**	**30**	**20 + 30 = 50**	**50**

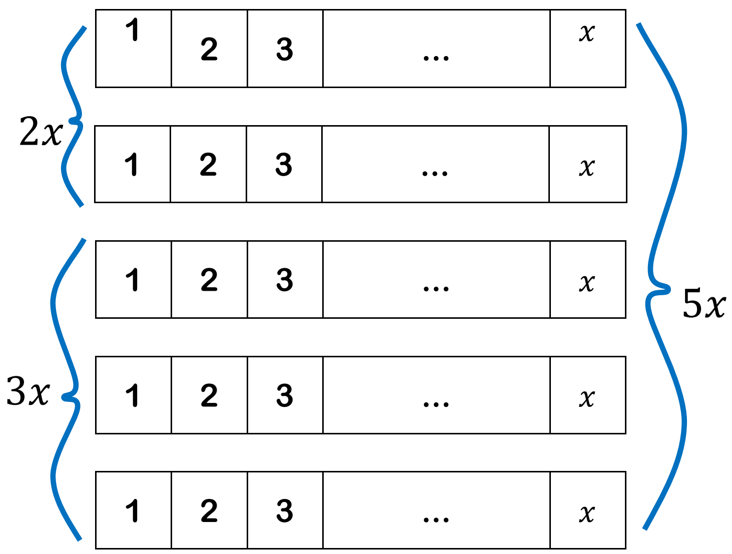
It always works!

Another way to think about this is physically in a picture. Let's take a variable number of squares and call it x. If we arrange them all next to each other in a row to form a long rectangle, we have a 1st square, a 2nd square, and so on all the way up to an x^{th} sqaure. The unknown bit is represented by the … in the middle section, as we don't know how many there are.

A rectangle containing x squares.

1	**2**	**3**	**…**	x

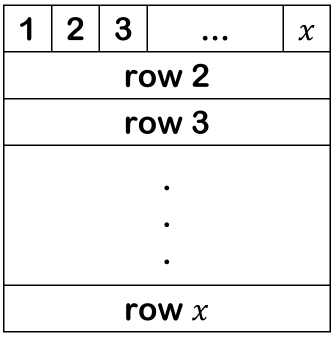
Now if we take 2 of these rectangles, and add another 3 of them, we will see that we get 5 identical rectangles.



We can do something similar when adding quadratic terms, this time in a grid. Let's look at

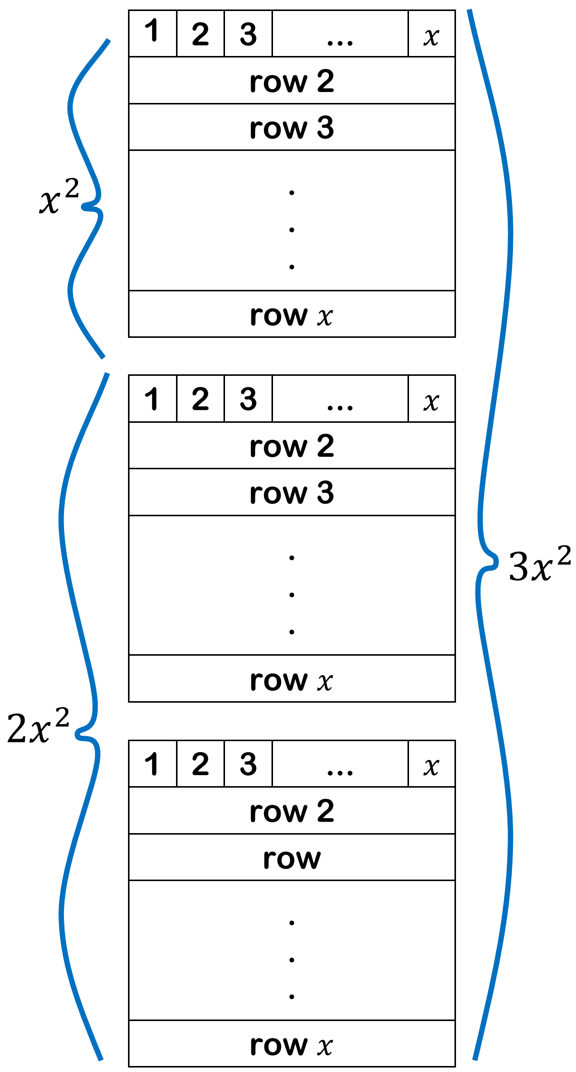
$$x^2 + 2x^2 \equiv 3x^2$$

The first row of the grid has x squares as before. And we have x rows of x squares in this grid, so it has $x \times x$ squares in all which is x^2 squares.



We then take one of these x^2 grids, and then add 2 more, and it shows us clearly that

$$x^2 + 2x^2 \equiv 3x^2$$



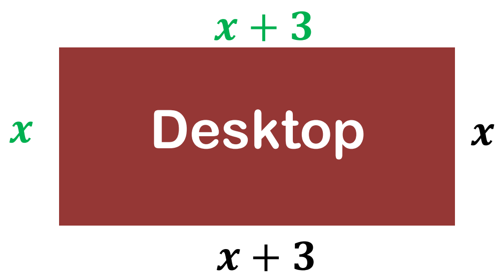
We can also be convinced simply by doing several substitutions as with liner expressions.

	x^2	$2x^2$	$x^2 + 2x^2$	$3x^2$
When $x = 1$	1	2	1 + 2 = 3	3
When $x = 3$	9	18	9 + 18 = 27	27
When $x = 5$	25	50	25 + 50 = 75	75
When $x = 10$	100	200	100 + 200 = 300	300

We could show that $4x^3 + 3x^3 \equiv 7x^3$ either in a substitution grid, or by taking a cuboid, made up of x^3 little cubes, to represent x^3. You then add 4 of these to 3 of these and see that you have 7 of these cubes in total. Have a try if you want to!

Now we will look at how this collecting like terms is useful in real situations.

Let's take a company that makes beautiful ornate writing desks. The top of the desk is always a rectangle (that happen to be measured in feet) with the length of the front 3 feet longer than the length of the side. The desktop is surrounded by a beautiful carved trim. How could one express the length of this trim as an expression. The side we could call variable number x and as the length is 3 feet more we can express the length as $x + 3$.



Because it is a rectangle, opposite sides are equal, so, in green, we can label the opposite sides the same.

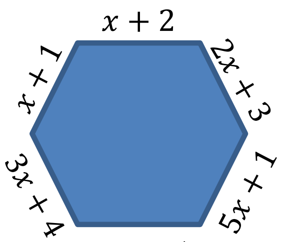
The length of the trim that goes around the edge of the desk top will clearly be the same as the perimeter of this rectangular desktop.

This can be expressed as:

$$x + x + (x + 3) + (x + 3)$$
$$= (x + x + x + x) + (3 + 3)$$
$$= 4x + 6$$

Note:
$x + x + x + x \equiv 4x$

It is easier to work with $4x + 6$ than the four separate expressions which make up the trim of the desk. We can add like terms in more complex situations too. Let' express the perimeter of the shape...

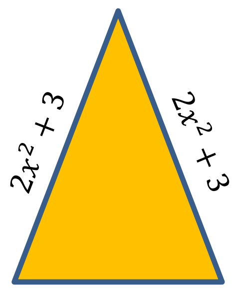


We can express the perimeter as:

$$(x + 2) + (2x + 3) + (5x + 1) + (3x - 5) + (3x + 4) + (x + 1)$$
$$= (x + 2x + 5x + 3x + 3x + x) + (2 + 3 + 1 + 4 + 1 - 5)$$
$$= 15x + 6$$

I hope this example makes it clear why expressions are useful. Rather than working out each side as 3 longer, or half as long as some other side, you work out the expression for the perimeter and can then substitute in several different values of x as needed.

Here's one that is "beyond" linear! Show that this triangle is isosceles and express its perimeter.



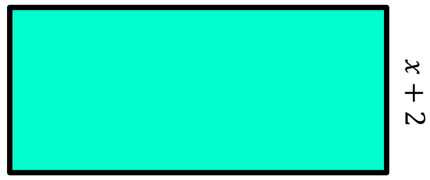
A triangle is definitely isosceles if it has either 2 equal angles or 2 equal sides. $2x^2 + 3 \equiv 2x^2 + 3$**, they are always the same, so these two sides are always the same, so we have an isosceles triangle.**

Now its perimeter is:

$$(7x^3 + 3x - 1) + (2x^2 + 3) + (2x^2 + 3)$$
$$= 7x^3 + (2x^2 + 2x^2) + 3x + (3 + 3 - 1)$$
$$= 7x^3 + 4x^2 + 3x + 5$$

Finally in this ladder of expressions, we will introduce the next ladder.. which is called brackets, and could be equally well named multiplying terms & expressions:

Find an expression for the area of this rectangle:



The area of a rectangle is found by multiplying the base (here $x + 1$) and the height (here $x + 2$).

So we express the area of this rectangle:

$$(x + 1)(x + 2)$$

And now we have two linear expressions (in brackets) to multiply, which is what ladder t is all about... Brackets!