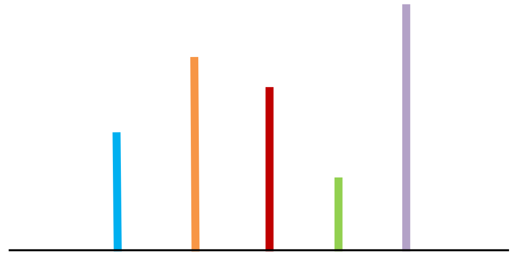
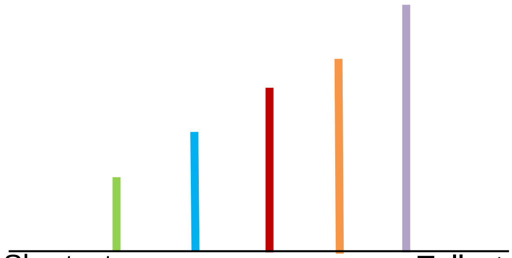
j) Size

j)
Size

Step 1) Comparing Lengths



Put these 5 lines in order of length, smallest on the left and longest on the right.



Shortest Tallest

These lines are upright so we can call the length, height, and the smallest is the shortest, and the longest is the tallest.

Which is the longest, a picture frame, a river, or the distance between two planets?

There is no river (on Earth) that is longer than the distance between two planets, so the distance between two planets must be longest. There is no river shorter than a picture frame, so the picture frame must be the shortest.

Step 2) Metric Length Units

Which of these measurements goes with which situation?

1) 18mm	a) The distance between Cambridge & Royston
2) 18cm	b) The width of a coin
3) 18m	c) The width of a motorway
4) 18km	d) The height of a hair dryer

The distance between 2 towns is clearly the furthest, so that is 18km. And the coin is clearly the smallest, so that is 18mm.

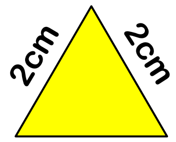
Of the remaining 2, a hairdryer is smaller than a motorway, so that must be 18cm tall, and the motorway must be 18m wide.

1) 18mm	a) The distance between Cambridge & Royston
2) 18cm	b) The width of a coin
3) 18m	c) The width of a motorway
4) 18km	d) The height of a hair dryer

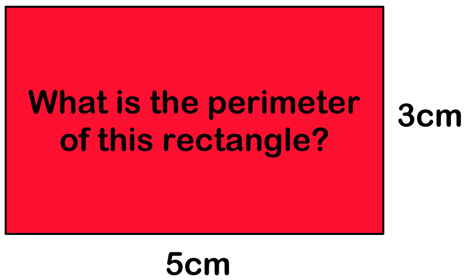
Step 3) Perimeter

The perimeter is the total length around the sides of a shape.

For example this equilateral triangle has sides 2cm, 2cm & 2cm,

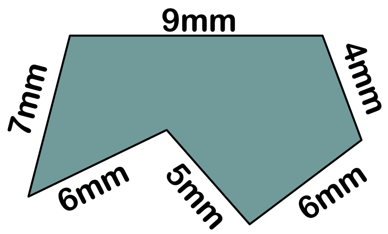


so it's perimeter is 2 + 2 + 2 = 6cm.



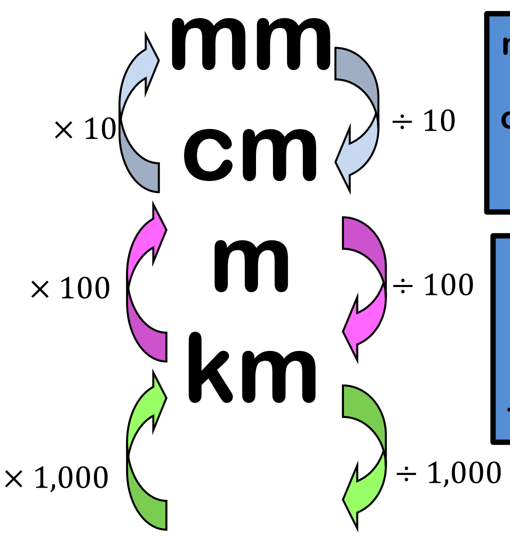
Because the opposite sides of a rectangle are equal, the sides are 5cm, 5cm, 3cm, & 3cm. The perimeter is the sum of these,
5cm + 5cm + 3cm + 3cm = 16cm

How about this very irregular hexagon that is not drawn to scale?



Perimeter is 7 + 9 + 4 + 6 + 6 + 5 + 6 = 43mm

Step 4) Metric Length Conversions



mm = millimetre

cm = centimetre

km = kilometre

1cm = 10mm

1m = 100cm

1km = 1,000m

This line is 8cm long. How many mm is that?

8cm

First check it with your ruler. Now 1cm = 10mm, and when going from larger unit to a smaller one, we multiply. So $8cm = (8 \times 10)mm = 80mm$

8cm

80mm

It may seem funny that when we move from a larger measure (like cm) to a smaller one (like mm) we multiply. Often to go from bigger to smaller we ÷ rather than ×. But remember we are only changing the units we measure with, not the actual size. If we are keeping the same length line, and we move to a smaller unit, we must fit more of them in, so we have to ×.

This is a scale drawing of a railway track that is 5,000metres long. How many km is that?

5,000m

Now 1km is 1,000m, and going from a smaller unit to a larger one we divide.

$$5{,}000m = (5{,}000 \div 1{,}000)km = 5km$$

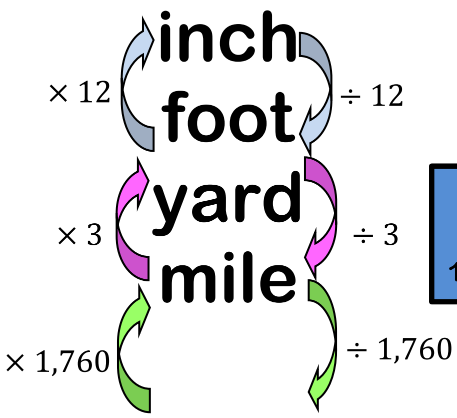
5,000m

5km

Step 5) Imperial Lengths

Many years ago, before time began (well actually any time before the 1960s) most people use old measures called imperial measure. They were based around things like the average length of a persons foot or hand, so everyday folk could use them by using their hands of feet to measure things. To convert between, say an arm and a leg, was never going to give a nice round number like 10, hence why we went metric. The main imperial units for measuring length were inches, feet, yards, and miles.

Interestingly for some reason, we never quite made it as far as making road signs metric, so the UK still uses miles on the road (most other places use kilometres)!



" = inch

' = foot

1' = 12"
1 yard = 3'
1 mile = 1,760yards

Just as with metric measurements, you multiply or divide to change between the units, but it isn't nice multiples of 10, so the calculations can be a bit harder.

How many yards is 24 feet?

Following the pink arrow above, we ÷3,
24 ÷ 3 = 8 yards.

Or, to put it another way…

1 yard = 3'
multiply both sides by 8
8 yards = 24'

Mixing Metric & Imperial

$$1'' \approx 2.5cm$$
$$1'' = 2.54cm$$

$$1' \approx 30cm$$
$$1' = 30.46cm$$

$$1mile \approx \frac{8}{5}km$$
$$1km \approx \frac{5}{8}mile$$
$$1mile = 1.609344km$$

We may also need to change between metric and imperial measures. We can simply multiply or divide both sides of the conversion equation by the same number to get there.

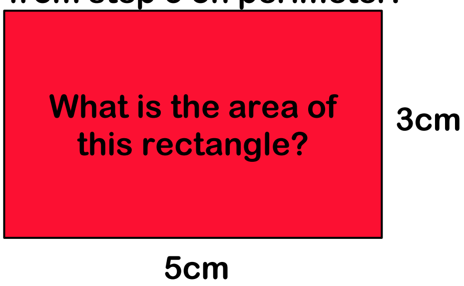
Approximately how many inches is 18''?

Using $1'' \approx 2.5cm$
x18 on sides you get
$18'' \approx 45cm$
or with the exact (but more awkward decimal) conversion
$1'' = 2.54cm$
x18 on sides you get
$18'' \approx 45.72cm$

Step 6) Area of a Rectangle

The formula for the area of a rectangle is
Area = Base x Height
A = bh

We can use this to find the area of the rectangle from step 3 on perimeter.

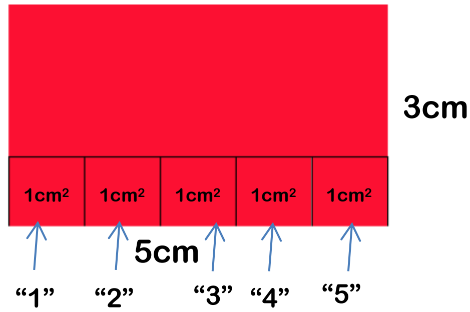


Area = base x height
= 5 x 3
= 15 cm^2

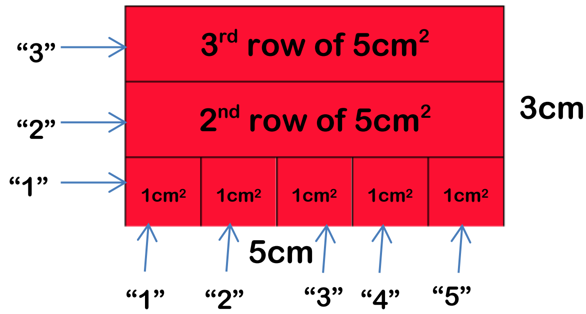
This is the "answer,"
but what-on-earth does it mean?

We measure area in unit squares (eg cm^2 or mm^2). So as the sides are in cm, in this case the area would be in cm^2 or cm squares.

The bottom row of this rectangle could fit exactly 5 cm^2 because it is 5cm wide.



And because it is 3cm high, we can fit 3 rows of 5cm^2 in this rectangle.



So in total we have 3 x 5cm^2 = 15cm^2

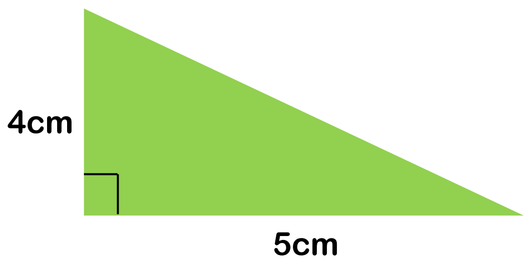
We can do this for any rectangle, and we will always get the width number of cm^2 in the bottom row, and the height number of rows. So we can see that any rectangle will fit base x height number of cm^2 inside it.

7) Area of Polygons

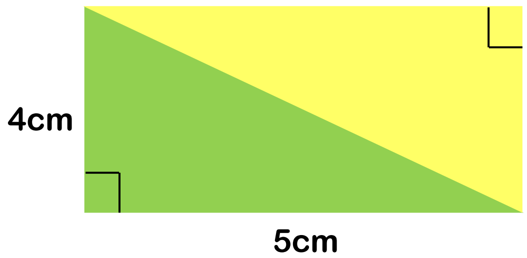
In this step we will deal with the area of triangles, parallelograms, and trapeziums.

A triangle can be thought of as half of a rectangle, so it is not surprising that you get half the expression for the area of a rectangle.

We'll look at a right angled triangle first.



Let's flip the green triangle over and make it yellow, and see how the two triangles (green & yellow) fit together!



You can see that these two identical (congruent!) triangles make a rectangle of base 5cm, and height 4cm, whose area is $20cm^2$. Crucially this means that each of the two triangles has ½ the area of the rectangle.

You could do this for any right angled triangle so the area of a triangle is ...

Area = $\frac{1}{2}$ x Base x Height

OR

$A = \frac{1}{2}bh$

OR

$$A = \frac{bh}{2}$$

And now using the formula in the above example.

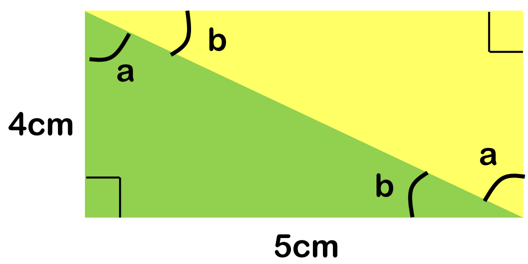
$A = \frac{1}{2}bh$

$= \frac{1}{2} x 5 x 4$

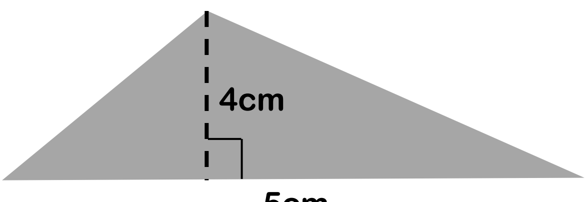
$= \frac{1}{2} x 20cm^2$

$= 10cm^2$

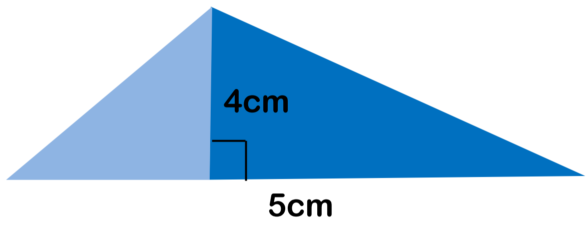
Note for students who have covered several steps of angles. Because the angles in a triangle add up to 180^0 and this is a right angled triangle, we know that a + b = 90^0 within the green triangle, and as yellow and green triangles are the same, you can see all four corners are a right angle. Similarly the opposite sides are the same, and hence we have a rectangle as required!



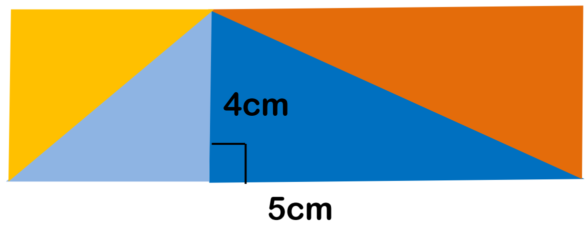
What happens if it is not a right angled triangle?



How do we know this would still be half of a rectangle of the same width & height? Let's split the grey triangle it into two right angled triangles, one light blue and one dark blue.



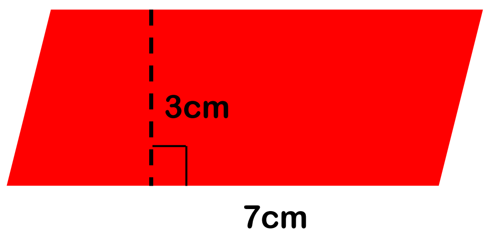
We can copy the light blue and dark blue triangles, flip them round, colour them light orange and dark orange, and put them all together to make a rectangle.



You can see that the light orange and light blue triangles are exactly the same size, and so the same area. The same is true of the dark orange and dark blue triangles. So the blue triangles together, must have half the area of all four triangles, and all four triangles make a rectangle of base 5cm, and height 4cm, which gives a rectangle of area $20cm^2$, and hence a grey/blue triangle of area $10cm^2$.

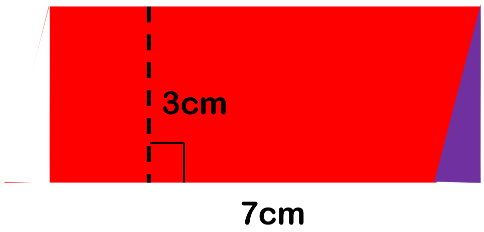
So our triangle formula works both for right angled triangles, and in fact for any triangle, as long as we measure the height at right angles to some "base" side – because we can use that height line to split it into two right angled triangles.

The area of a parallelogram also uses this idea of moving around triangles.



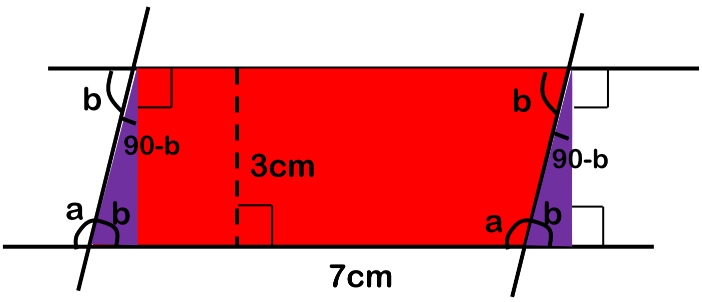
Amazingly, this parallelogram, has the same area as a rectangle with the same base and height! So it's area is 3 x 7 = $21cm^2$

Let's we cut off a purple triangle, with top vertex perpendicular to the base. We then move it to the other end and see that it fits exactly, creating a rectangle with the same base, height, and area as our parallelogram.



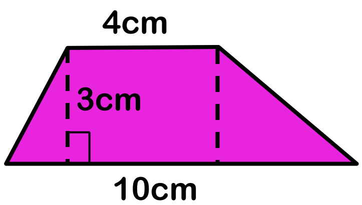
So the area of a parallelogram is...
Area = base x height
OR
A = bh

This is enough to understand the parallelogram area formula, but to prove it fully the purple triangle fits, we need some advanced steps from angles, called the parallel line angle rules. We simpy extend all the parallel lines....

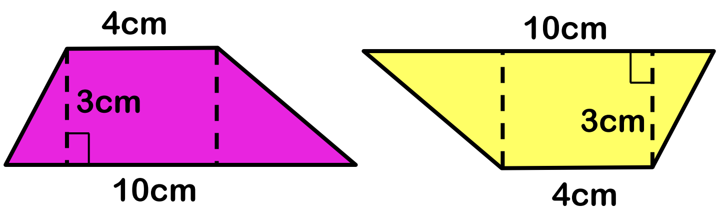


Using the parallel line angle rules we can show that all the angles in the purple triangles fit just as well on the right of the parallelogram (making a rectangle) as they did on the left!

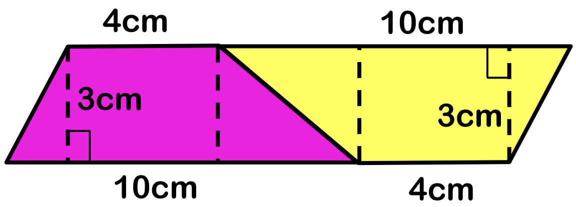
And finally a trapeizum. Let's look at one with top 4cm, bottom (base) 10cm, and height 3cm.



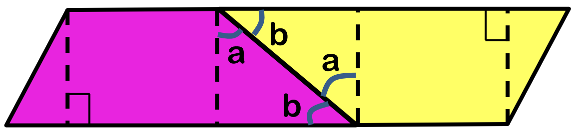
If we take a second copy of this trapeizum, flip it over, paint it yellow like so...



We can put them together to make a parallelogram.



It has the same height, and it's base is the length of the top and the bottom (of the original trapezium) added together.

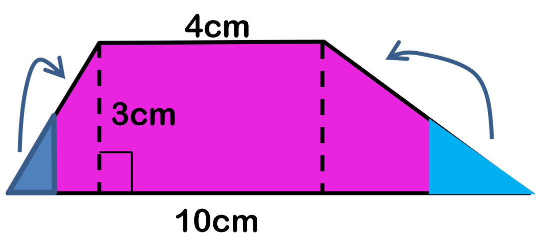


Please not how the angle's a & b (because the dahsed line forms a right angled triangle) have sum 90°. So a & b fit together, along with the right angle, fit together to form a straight line (180°) on the top and bottom of the parallelogram created from two trapieziums (or is it trapizii!? - discuss).

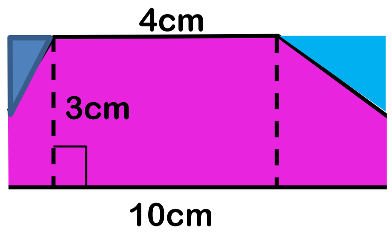
So the area of this double trapezium is 3(4+10) or in general height(top + bottom). Because it is two of the original trapezium put together, we just halve it to find the area of each trapezium.

$$Area_{Trapezium} = \frac{1}{2}(top + bottom)height$$

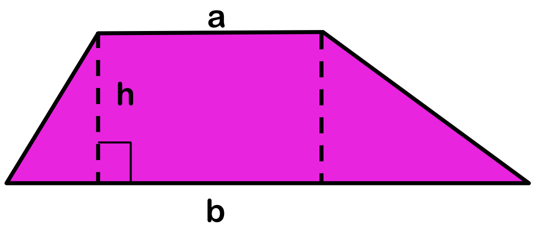
Another, but slightly more complicated way to do this is to make a small triangle in each of the bottom two vertices, half the size of the triangles created by the two vertical dropped from the top two vertices and perpendicular to the base (dashed lines).



These then flip over and fit into the gaps to make a rectangle. Because the smaller and larger triangles are enlargements of one another (similar), the fliped over angles connect to make a right angle, and the hypotenues fits exactly as the smaller triangle was exactly half of the larger.



So we know that the height of the rectangle we have created is 3cm – but what is the width. Well the top line of the two small blue triangles, were the edges of bottom of the base of the original trapezium. So the top and bottom of this rectangle are made up of the bottom and top edges of the original trapezium. As the top and bottom of a rectangle have equal length, we can just add the total of the bottom and top of the oringinal trapezium and halve it. Here 4 + 10 = 14, then ½ of 14 is 7, so the base is 7cm. Hence the area if 7 x 3 = 21 cm^2

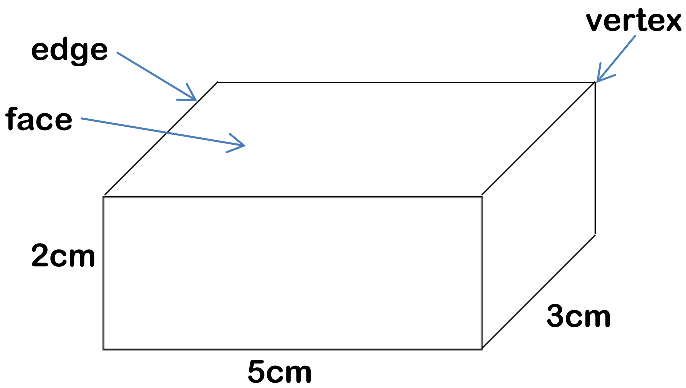


In general, the area of a trapezium is:

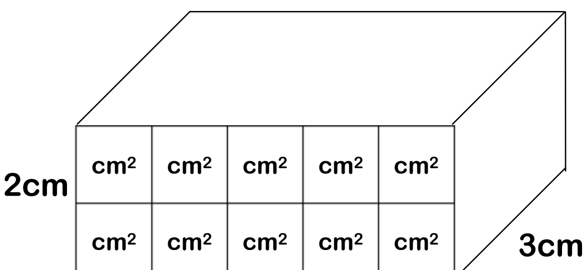
$\frac{1}{2}(a+b)h$ or $\frac{a+b}{2}h$

8) Volume of a Cuboid

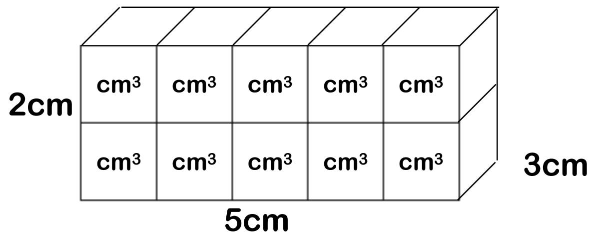
A volume is a measure of the size of a 3D shape, like a tupperware, a football, or a bath. It is measured as how many units^3 (in the example below this is cm^3, or centimetre cubes) we can fit into the shape. The easiest shape to measure the volume of is a cuboid...



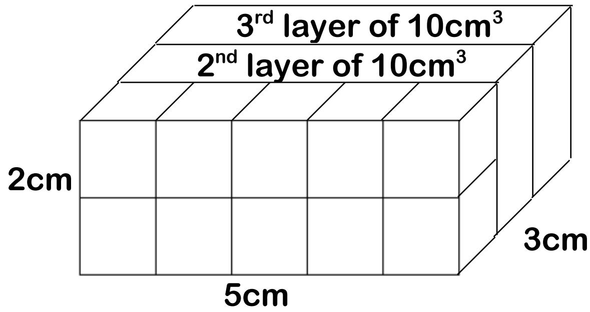
The area of the front face is 10cm^2.



If we take a layer 1cm deep then each cm^2 of the front face, will become a cm^3



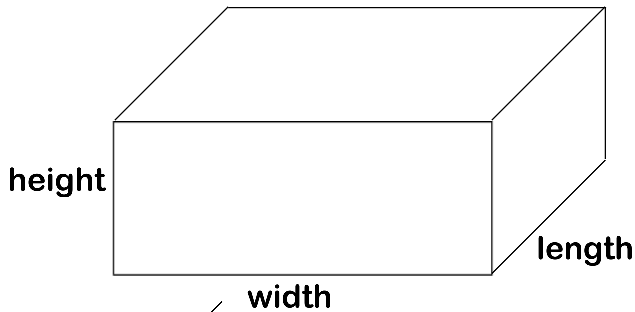
So the front 1cm deep layer has volume 10cm^3



Because the shape is 3cm deep, we get 3 of these 10cm^3 layers, so the volume of the shape is 3 x 10cm^3 = 30cm^3

In general for the volume of a cuboid is:

Volume = width x height x length
or
V = whl



9) Capacity

We have many clever ways to measure the number of cm^3 that fit into a 3D shape, but something like a bathtub would be very hard to measure! Having said that, if you took a bath full of water, and poured the water into an empty cuboid you could measure how high the water filled it up, then you'd know the volume of the water, and hence the volume of the bath! We can the volume of a liquid capacity, and it is measured in millilitres (or ml). A ml is basically the same as a cm^3 of water (which happens to weight 1g too – the metric measures all fit together and are based around water).

A litre of water is a cuboid 10cmx10cmx10cm, as this is 1,000cm^3 which is 1,000 ml.

1 Litre = 1,000 millilitres
1 l = 1,000 ml

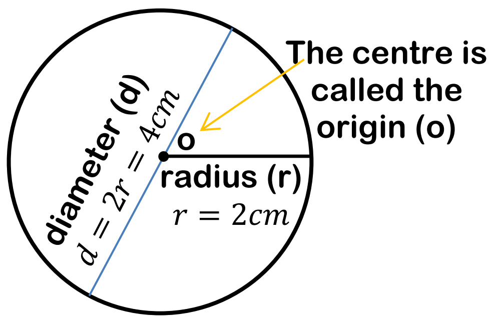
Match up the folllowing.

1) 7ml	a) A reusable bottle of water
2) 600ml	b) A bath
3) 23 million l	c) A teaspoon of medicine
4) 80 l	d) A lake

The teaspoon is smallest and the lake largest, so they must have capacities of 7ml and 23 million l respectively. Of the middle two, the bath is largest, so the bath must have capacity 80 l, and the teaspoon of medicine 7ml.

10) Circumference & Area of a Circle

Measure the radius & diameter of this circle.



Circles are so special, that unlike any other shape, they have their own unique name for their perimiter. The perimiter of a circle is called it's **circumference**!

Take a piece of string and wrap it around the circumference, and then measure the string to find the length around the edge of the circle.

Try this for a few different circles, and see how many times the diamter (or radius) fits around the cricumference....

You should find that just over 3 dimaters (or just over 6 radii) fit around the edge of the circle.

In actual fact it is a very special constant number that tells us exactly how many fit... it is the amazing number pi which has it's own special symbol... π
$\pi = 3.14159265$ to 8dp, though in fact the decimal goes on... forever without repeating.
We usually just use pi rounded to 2dp $\pi = 3.14$ or even sometimes just $\pi = 3$

So the formula for the circumference (a word that just means perimeter, but can only be used with circles), C of a circle is $C = 2\pi r$ or $C = \pi d$

With out above circle of radius 2cm and diamter 4cm, we can use either formula, and get the same answer. We'll use $\pi = 3.14$

With radius: $r = 2$ and

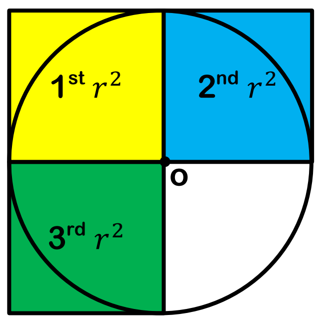
$$C = 2\pi r$$
$$= 2 \times \pi \times 2$$
$$= 4\pi$$
$$= 12.57\ (2dp)$$

or

With Diameter: $d = 4$ and

$$C = \pi d$$
$$= \pi \times 4$$
$$= 4\pi$$
$$= 12.57\ (2dp)$$

We can do something similar for the area of a rectangle. Because area is measured in units^2 we will work with radius^2 or r^2. Lets take 3r^2 (3 r-squares) and see if they will fit inside the circle.



You can do this yourself, or used the A4 version of this in our resources folder. You'll see that when you cut out the yellow, blue and green parts of the r^2 that our outside the circle out, you can cut them up and fit them into the uncoloured part of the circle. There will be a little bit of space left over. This means that a little more than $3r^2$ fit into a circle. In fact it is πr^2. So the area of a circle is $A = \pi r^2$

In our example then, where $r = 2$

$$A = \pi r^2$$
$$= \pi \times 2^2$$
$$= 4\pi$$
$$= 25.13\ (2dp)$$

There's a really nice way to remember these formulas!

If you prefer $C = 2\pi r$ then this song to the tune of nursary rhyme twinkle twinkle little star.

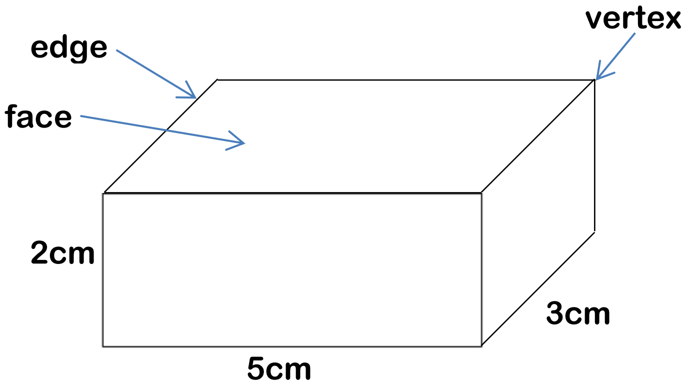
Twinkle, twinkle, circle star
Circumfernce is two pi r.
One more thing which must be shared.

Area is pi r squared.
Twinkle, twinkle, circle star
Circumfernce is two pi r.

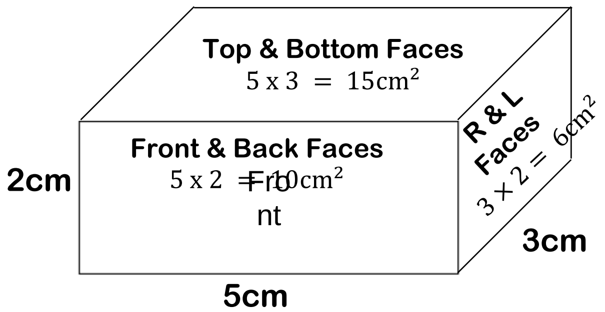
If you prefer $C = \pi d$ you can repeat...
Cherry pie's delicious
Apple pies r 2

Step 11) Surface Area

Surface area is the area of all the faces of a 3D shape. Let's look at a cuboid.



We work out the area of the faces in pairs, as opposite faces are the same size.

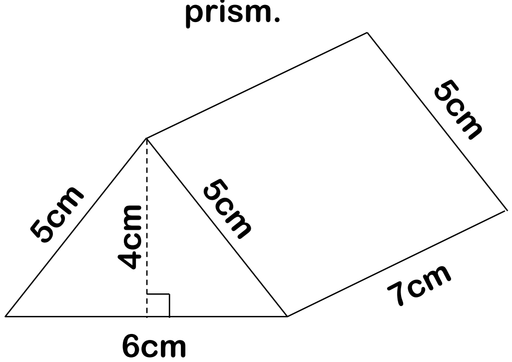


So in total we have

top + bottom + right + left + front + back
$= 15 + 15 + 6 + 6 + 10 + 10$
$= 62cm^2$

However complex the shape – we break it up into its faces and work the area of each out seperately.

Let's find the surface area of this triangular prism.



Front & back faces $= \frac{4\times6}{2} = 12cm^2$ (each)

Right & left faces $= 5 \times 7 = 35cm^2$ (each)

Bottom face $= 6 \times 7 = 42cm^2$

Total surface are $= 2 \times 12 + 2 \times 35 + 42 = 136cm^2$

12) Volume of a Prism

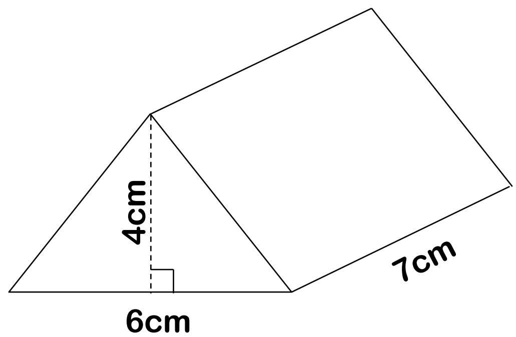
A prism is a 3D shape, that will leave you with the same 2D shape cutting anywhere along the length of it.

A cuboid (a rectangular prism), and a cylinder (a circular prism) are both types of prism. Some mathematicians argue that the face you make when cutting the shape (called the cross-section) must be a polygon – but here we'll allow it to be any shape.

In fact one nice way to remember the word prism is with the word prison if you imagine the old style (and deeply cruel and inhumane) prison food of bread and water. As whever you cut a loaf of bread (if it has been perfectly baked) you get the same shape.

Because each slice has the same area – we can do the same things we did with a cuboid, taking 1cm deep slices.

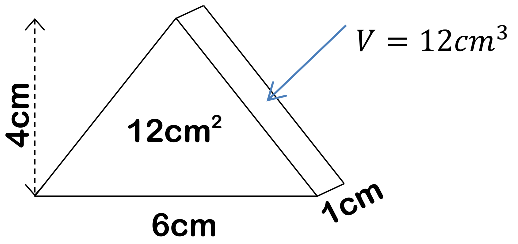
Let's look at a triangular prism



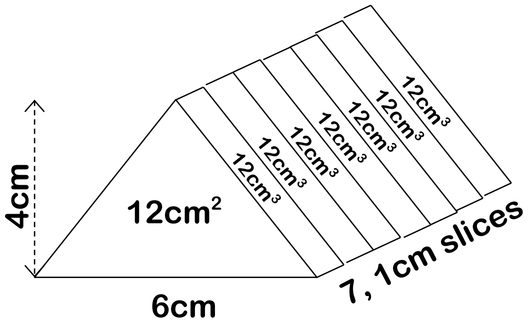
Firstly let's calculate the area of the front face. This will be the area wherever we cut a cross-section (by cutting across the shape).

$$Area = \frac{4\times6}{2} = 12cm^2$$

Now if we take a layer or slice exactly 1cm deep, then it's volume will be $12cm^3$.



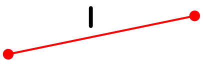
Now because it is 7cm deep – there will be 7 slices each of volume 12cm³

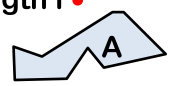


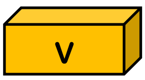
So the overall volume of the trianglular sprism will be $7 \text{ x } 12 \ = \ 84 \text{ cm}^3$

13) Units of Length, Area & Volume

In this step we will be looking at how we can see whether a formula is one to calculate an area, a length or a volume without knowing the specific type of shape. We can tell from the formula whether it is 1 dimensional (1D) and so a length, 2 dimensional (2D) and so an area or 3D and so a volume.

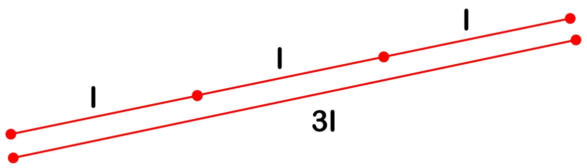
Let's look at a length l 

an area A 

and a volume V 

Firstly we can say that if we multiply any of these by a constant we will still have a length area or a volume as we started, just a bigger (or smaller) length, area or volume. For example if we take 3l (or three lots of the length l we started with).

3l = l + l + l

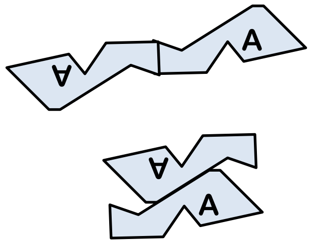


Which is clearly a length so

Constant x Length = Length

Now lets look at doubling an area.

2A = A + A



However we put our 2 As together we will still have an area so

Constant x Area = Area

Similarly with volumes

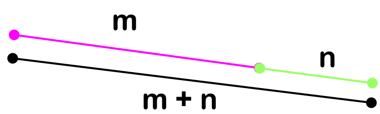
Constant x Volume = Volume

Try putting some cubes of volume 1cm³ together to make something other than a volume (without the use of a hammer!)

This will be true even for interesting constants like 7l (still a length), πA (still an area) and -3.2V (still a volume – though conceptually a 3.2 V sized holes in the sand rather than 3.2 piles of sand.

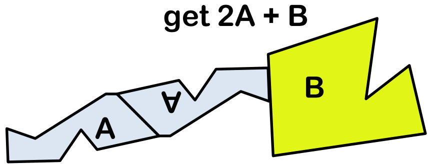
Similarly adding fifferent lenghts, areas or volumes will keep it as a length area or volume.

So lengths m & n added gives m + n



Which is clearly still a length ☺

If we add two As (from before) and a new area B we get 2A + B



And this is clearly still an area.

So
Length + Length = Length
Area + Area = Area
Volume + Volume = Volume

Finally, what happens if we times different ones together!

Length x Length = Area

Looking back at all our formulas for the areas of shapes from steps 6 & 7 we can see that we always multiplied two lengths to make an area.

1cm x 1cm
= 1 x cm x 1 x cm
= 1 x 1 x cm x cm
= 1 x cm^2
= $1cm^2$

This was especially clear in step 6 looking at a rectangle.

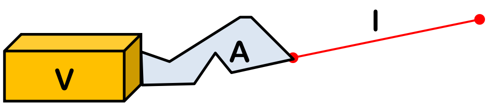
Similarly looking at cuboids and prisms, to make a volume, you first find the area of the front face, then realise that a 1cm deep layer will be the same number of cm^3 as you had cm^3 on the front face. And then multiplying by the number of layers like this. In other words multiplying a 2D area by a 1D length makes a 3D shape (ie a volume).

$1cm^2$ x 1cm
= 1 x cm^2 x 1 x cm
= 1 x 1 x cm^2 x cm
= 1 x cm^3
= $1cm^3$

So overall

Length x Length = Area
Area x Length = Volume

Now try adding together a length, and area and a volume say l + A + V



They're all so different, they just won't go together in a way that makes any sense!

Let's put this all together with a couple of abstract examples.

If m & n are lengths, and q is an area.

Is 4m + 7n a length an area or a volume?

4m = Area
7n = Area
Area + Area = Area
so 4m + 7n is an area

Let's try $\frac{\pi m^2}{2}$
now q^2 is an area
so πm^2 is just a bit more than $3.14m^2$, in a other words an area. And dividing by 2, is just halving. This is a constant, half of an area is an area.

So $\frac{\pi m^2}{2}$ is an area.

Finally we'll look at $\frac{7m^2n}{Q}$
m^2 is a length times a length, so it is an area.
An area (m^2) times by length n then gives a volume.
And 7 times a volume is still a volume.

So $7m^2n$ is a volume

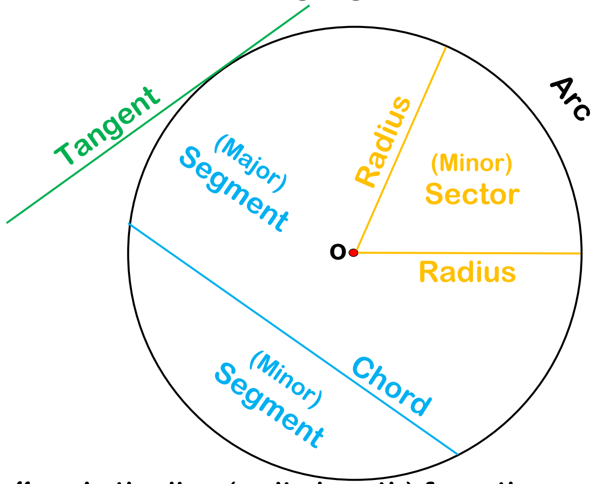
BUT, what happens when we divide a volume by area Q?

Area x Length = Volume
so
Length = $\frac{Volume}{Area}$

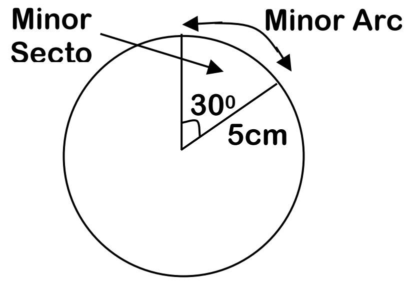
So $\frac{7m^2n}{Q}$ is a length!

Step 14) Sectors & Segments of a Circle

Firstly you need to know some important circle language...



Radius: Is the line (or its length) from the centre of a circle to a point on its circumference (Plural is radii).
Tangent: Touches a curve at exactly one point.
Chord: Is the line between two different points on the circumference of a circle.
Segment: Is one of the two areas created when a circle is separated by a chord.
Sector: Is the slice (shaped like a pizza slice) when a circle is cut into two pieces using two radii.
Arc: An arc is a part of the circumference of a circle between two points.
Major/Minor: The major segment/sector/arc is the larger part. The minor segment/sector/arc is the smaller part.
Remember that a circumference can be thought of as two radii in a straight line, or as a special chord that goes through the centre (or origin) of a circle.



Finding the area of sectors and the length of arcs is really just a case of finding the fraction of the circle that you sector is.

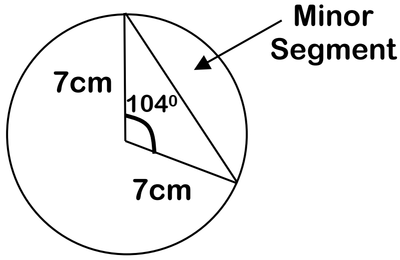
So this one (above) has a 30^0 minor sector, and as the whole circle is 360^0, this is $\frac{30}{360}$ of the circle which is $\frac{30}{360} \div \frac{30}{30} = \frac{1}{6}$. So we simply need to find $\frac{30}{360}$ (or $\frac{1}{6}$) of the area of the circle.

Area of Minor Segment = $\frac{30}{360} \times \pi \times 5^2 = 6.54cm^2(2dp)$

Similarly to find the length of the minor arc we would find $\frac{30}{360}$ (or $\frac{1}{6}$) of the circumference of the circle.

Length of minor Arc = $\frac{30}{360} \times 2 \times \pi \times 5 = 2.62cm(2dp)$

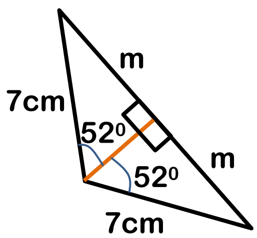
Now let's look at a question about the area of a segment.



Firstly we find the area of the minor sector

Area of minor Sector = $\frac{104}{360} \times \pi \times 7^2$

Then we will focus in and look at the triangle created by the two radii and the chord. This is an isosceles triangle, and we will cut the 104^0 angle in half to make two identical right angled triangles, each with a 52^0 angle at the centre of the circle.



The orange line that bisects the triangle also bisects the chord. So m is the opposite of a right angled triangle whose hypotenuse is 7cm. We can find both m, and hence the chord length, as well as the length of the orange line and hence the area of this triangle.

$$m = 7cos52$$

so the chord length is double this, 14cos52

the orange line is 7sin52

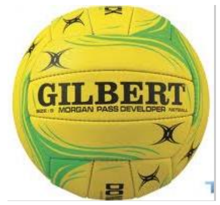
hence the area of the triangle is $\frac{7\text{sin}52 \times 14\text{cos}52}{2}$
= 49sin52cos52

So the are of our segment is the area of the sector minus the area of the segment which is…

Area = $\frac{104}{360} \times \pi \times 7^2 - 49\text{sin}52\text{cos}52$

To summarise, with segments, we use the isosceles triangle formed by the chord and two adjacent radii, and split it in two to form two congruent right angled triangles. We can then use Pythagoras and/or trigonometry to find the length of the chord (in two equal halves) and the area of the isosceles triangle. From this we can find the area of the segment using the fact that the segment area plus the isosceles triangle area = the sector area.

15) Volume & Surface Area of a Sphere



This netball has radius 14cm, what is it's volume?

Once upon a time Archimedes solved a problem a bit like this to find the volume of a gold crown. He dunked it in a full bath and measured the volume of the water that tipped over the edge of the bath – eureka!

But now we know how to find the volume of a sphere.

Volume of a Sphere $= \frac{4}{3}\pi r^3$

You can test this roughly in real life with play dough. The 3 on the bottom of the fraction and the π almost cancel one another so that $\frac{4}{3}\pi r^3 \approx 4r^3$. So we can take 4 r-cubes, cubes with the 3 dimensions being r (here 14cm) and then squish the four cubes together and roll into a ball to be our sphere shape. The diameter of this ball should be 2r (here 28cm).

We can try this out with smaller spheres but we'd need a lot of playdough for the netball. We can prove this formula when we learn about volumes of rotation in A-level maths using a technique called integration.

But for now let's find the volume!

Volume of a Netball $= \frac{4}{3}\pi 14^3$

Also using volumes of rotation you can find a formula for the surface area of a sphere.

Surface Area of a Sphere $= 4\pi r^2$

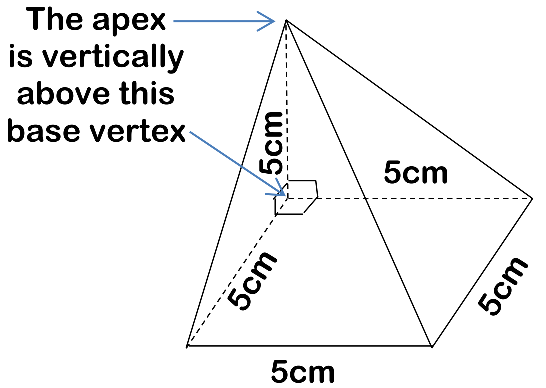
You could test this out with a netball, by taking 12 r-squares (as $4\pi r^2 \approx 12r^2$), that is 12x14cm squares and sticking them onto the net ball and seeing if they fit.

Or using the formula we calculate that the

Surface Area of a Netball $= 4\pi 14^2$

16) The Volume & Surface Area of a Pyramid & Cone

Construct 3 of these (5cm) square based pyramids, where the apex is vertically above one of the sqaures vertices.



Can you fit the three pyramids together to make a cube (with edges all of 5cm)?

If these three identical pyramids have the volume of a cube, then each pyramid must have volume one third of the cube.

So the volume of a pyramid from a cube is:

$$Volume = \frac{1}{3} \times Area\ of\ Base \times Height$$
$$V = \frac{1}{3} \times B \times h$$

In fact this works for all pyramids – we haven't proved it fully here, but we have givena good sense that it is true.

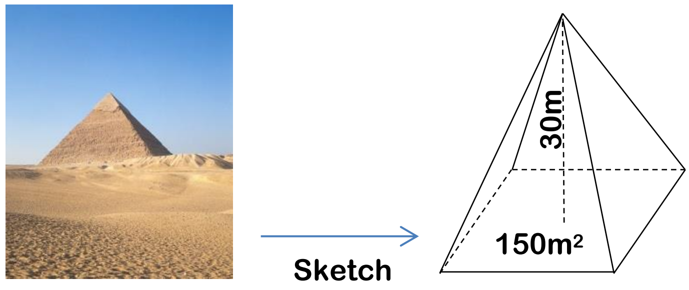
A cone is a circular based pyramid the volume of a cone, with height h and base radius r is:

$$V = \frac{1}{3} \times \pi r^2 \times h$$

In short if you think of pyramids and cones as one third of the respective cuboid or cylinder, you can easily remember the formula for and get a sense of what the volume is. Where this 1/3 comes from will be understood much more deeply with volume of rotation in the area of A-level maths called integration – it is quite literally a revolution in finding areas and volumes with complex shapes.

Here are two examples…

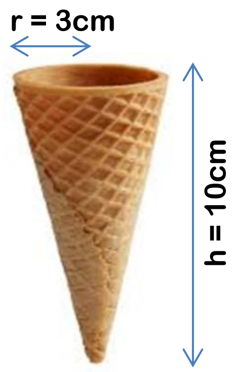
This pyramid has base 150 m² and perpendicular height 30 m. What is its volume?



$$Volume = \frac{1}{3} \times Area\ of\ Base \times Height$$
$$= \frac{1}{3} \times 150 \times 30$$
$$= 1{,}500 m^3$$

The surface area of a pyramid can be calculated by finding the area of all the polygons that are its faces.

What volume of ice cream can you fit within this ice cream cone (without any piled on top!)?



$$V = \frac{1}{3} \times \pi r^2 \times h$$
$$V = \frac{1}{3} \times \pi \times 3^2 \times 10$$
$$V = 30\pi$$

The surface area of a cone can be found using the formula…

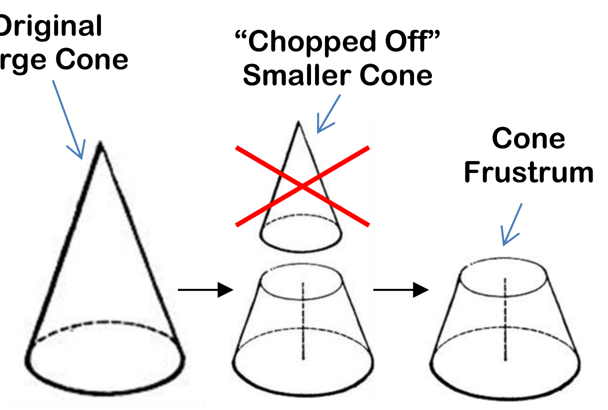
$$Surface\ Area\ of\ a\ Cone = \pi r^2 + \pi r\sqrt{h^2 + r^2}$$

The first part of this formula is just the area of the circular base of the cone πr^2 and the more complex part is the area of the curved face around the cone (the wafer part of the ice cream cone), given by $\pi r\sqrt{h^2 + r^2}$

So the (non-overlapping) area of wafer needed to make this ice cream cone can be found using

$$Area = \pi r\sqrt{h^2 + r^2}$$
$$= 3\pi\sqrt{10^2 + 3^2}$$
$$= 3\pi\sqrt{109}$$

The final part of finding sizes with cones you need to understand is about something called a **cone frustrum**. This is the shape left when you chop a little cone off the top of a larger cone.



To find the volume of a cone frsutrum, first we find the volume of the large original cone. Then we find the volume of the smaller "chopped off" cone. Finally we find the difference between these two volumes which leaves us with the volume of the frustrum.

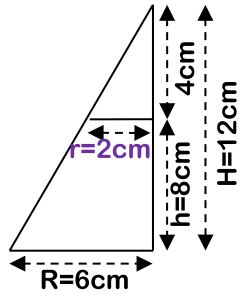
Find the volume of a cone frustrum, where a cone of height 4cm has been chopped off a cone who's original height was 12cm and original base radius 6cm.

<u>Large Cone's Volume</u>

$$V = \frac{1}{3}\pi \times 6^2 \times 12$$
$$V = 144\pi$$
or
$$V = 452.3893421cm^3$$

<u>Small Cone's Volume</u>

There is often a nice twist here in these questions (which can for many be the hardest part of frustrum questions) where some needed information is missing. They may only give one of the heights, or one of the radii, or perhaps they will give the diagonal height of the cone, but not the perpendicular height. Whatever information is missing, the solutions all rely on right angled triangles as the radii, the heights and the diagonal side of the cone will always form a right angled triangle. Depending on what information we do and don't have we can use either Pythagoras, trigonometry or (as in this case) similar triangle properties to find the needed, but missing information. Here the radius of the smaller cone is missing.



We know that 4cm is chopped off a 12cm high cone, so the frustrum must be 8cm tall. We know the radius of the large cone was 6cm, and that as we have two similar triangles here the ratio between the heights and radii must be the same, this ratio is $\frac{smaller}{larger} = \frac{4}{12}$ so

$$r = 6 \times \frac{4}{12} = 2cm$$

Now we have all the information we need!

Let's now find the volume of the small cone.

$$V = \frac{1}{3}\pi \times 2 \times 4$$
$$V = \frac{8}{3}\pi$$
or
$$V = 16.735516082cm^3$$

<u>Volume of the Cone Frustrum (Finally!)</u>

Volume of the Frustrum = large cone – small cone

$$V = 144\pi - \frac{8}{3}\pi$$
$$V = 141\frac{1}{3}\pi$$

or

$$V = 452.3893421 - 16.735516082$$
$$V = 435.6cm^3(1dp)$$