m) Understanding Constructions

1 2 3 4 5 6 7 8 9 10

m) Constructions

Step 1) Measuring and Constructing Lines

In the size and angles ladders we regular use a sketch, a type of drawing where the labels are accurate but the actual sides and angles are not what they say they are. A construction is a kind of drawing where the sides and angles are exactly what they say they are. So if it says 5.7cm that side must be exactly 5.7cm. If an angle is labelled as 45^0 then that angle must be exactly 45^0.

Let's try and measure how many cm long this line is.

To measure lengths you need a tool called a ruler. This is a straight edged piece of metal, plastic or wood with markings along the edge in a particular unit of length.

A Ruler

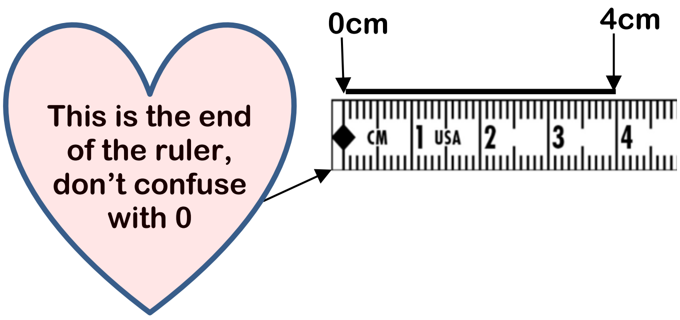
The ruler we are using here measures in cm and mm. The large lines going right across the ruler mark the cm, that is they are 1cm apart. The little lines are each 1mm apart.

The diamond at the end represents where to start measuring, in other words it represents a length of 0cm.

There is a little extra bit at the end of many rulers, so that if they get worn down over time a little bit it doesn't change the lengths you measure.

The question implies this might be a whole number of cm. Here's what to do…

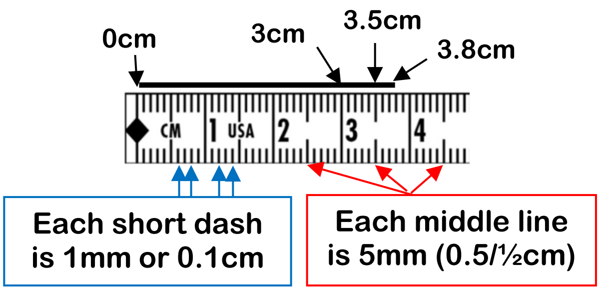
First place the 0cm amrk exactly at the start of the line you are measuring, lining your ruler up touching the line all the way along (the ruler should be parallel to the line). Then see at what point on the ruler the end of the line reaches.



Here you can see the line ends exactly over the 4cm mark. There are 4cm between the 0cm mark and the 4cm mark, and you can see that this is the same length as the line so the line is 4cm long.

Now let's try another line that is not an exact number of cm long.

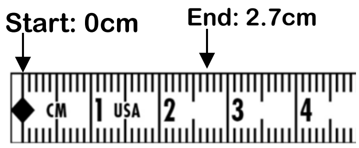
First put the end of the line on the 0 mark as before, and then make the ruler touch the line so both are parallel.



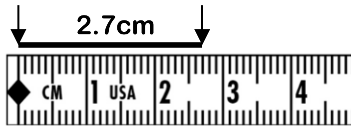
You can see that the end of the line goes past the 3cm mark. Now you can just count how many millimetres it goes past, which is 8, so this line is 3cm and 8mm, otherwise written as 3.8mm.

However it is very fiddley counting lots of millimetres so a quick trick is to know that the middle line at 5mm or ½cm is slightly longer. So this is 3mm past the middle line and 5mm plus 3mm is 8mm. Hence 3cm and 8mm.

Constructing a line is quite a similar process. If you want to construct a 2.7cm line, you need to line your ruler up where you want to draw the line, with the 0 at the start point and the 2.7 at the end point.



And then you simply draw the line along the edge of the ruler between those two points.



And when you move the ruler you have a 2.7cm line.

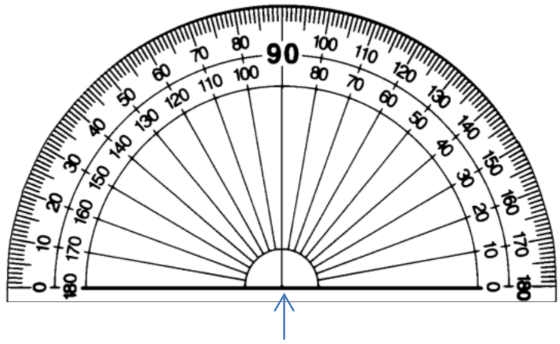
2.7cm

Step 2) Measuring & Constructing Angles.

An angle is a measure of turn, with a full turn being broken into 360^0 little mini turns called degrees. So a half turn, for example would be 180^0.

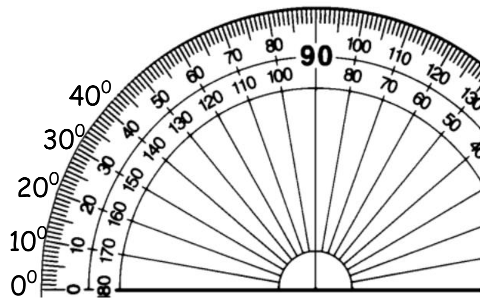
To measure angles we use a circular object called a protractor. You can get ones that are a full circle and have a scale all the way from 0^0 of turn to 360^0 of turn. Or a half circle that only covers 0^0 of turn to 180^0 of turn.

Here's a 180^0 (half turn) protractor. It is a semi-circle shape.

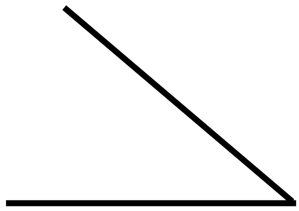


From the centre of turn (the cross) you can measure how far you have turned on the outer scale (if turning clockwise from the zero on the left) or the inner scale (if turning anti-clockwise from the zero on the right).

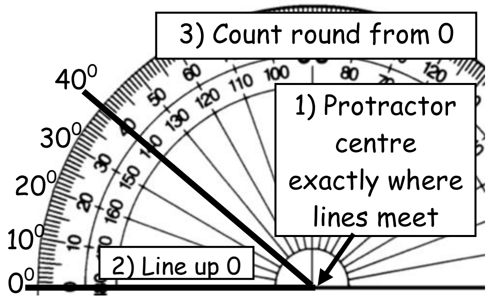
Below let's mark just every ten degrees (the slightly longer scale marks) but there is a little mark every degree so you can measure the turn very precisely.



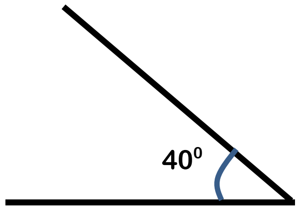
Let's measure the angle of turn between these two lines.



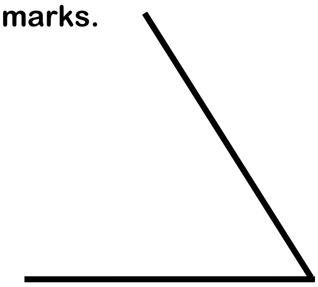
Firstly you put the centre of turn, that's the cross in the middle of the protractor, on the vertex which is the centre of turn between the two lines. You then line the zero up with the first line. Then counting round from 0 you see how many degrees you have turned to get to the second line.



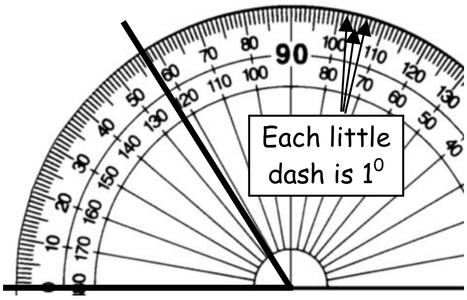
So the turn between these two lines is 40^0, we'll label it as such.



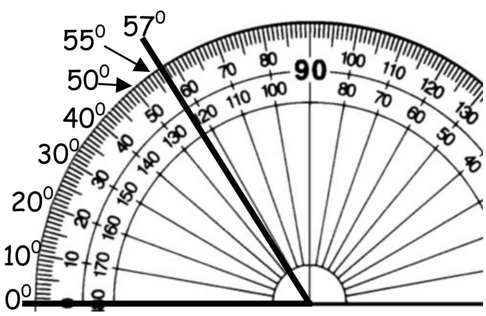
Now that example was a nice round numbered angle, a multiple of ten. Let's look at one where you have to use the little 1^0 measurement marks.



As before, we put the cross on the vertex at the centre of turn and line up the zero with the first line. It is important to realise that each little tiny dash marks 1^0 of turn. Each long dash marks 10^0 of turn, and the middle half-length dashes mark 5^0 of turn.

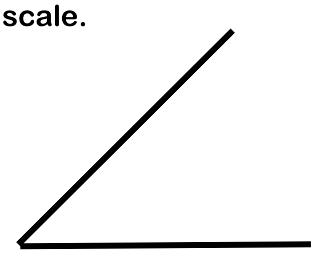


Finally we count round from the zero on the outer scale (as it is a clockwise turn from our starting line, to our finishing line). After we get to 50^0 (counting round 5 long 10^0 dashes) we have 7 little 1^0 dashes also to turn to the finish line.

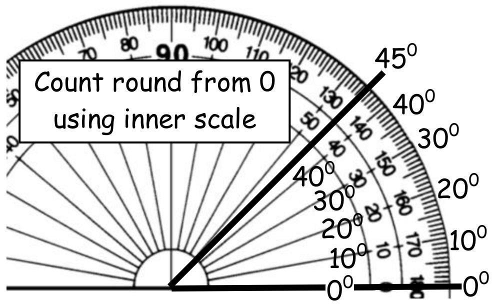


It is worth noting that we could have counted the 7^0 part (after we'd turned to the 50^0 mark) in a step of 5^0 to the medium length 5^0 dash and then another 2 little 1^0 dashes, making a total of 7^0 of turn after the 50^0 mark.

Let's try on counting anti clockwise, using the inner scale.

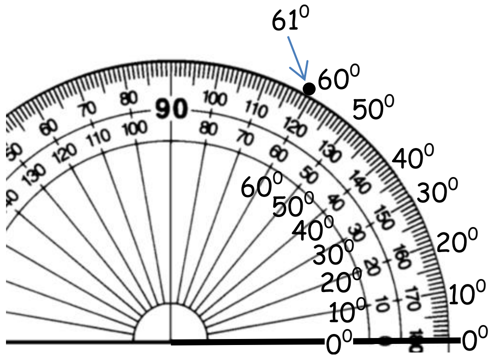


We still need to put the cross on the vertex of turn, but now the zero is lined up on the right side of this, which means the zero turn is on the inner scale (We can't start our turn at 180^0 if we haven't turned at all). Then we measure the turn counting anti-clockwise round the outer scale. This angle turns past the 40^0 dash (turning past 4 long 10^0 turn marks) and then ends on a middle mark, so there's another 5^0, hence 45^0 in total.

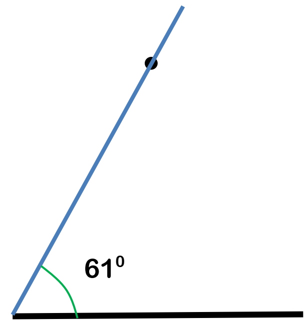


Finally let's try and construct an angle, to represent a turn from the first line to the second line of 61^0.

We need a first line, let's call this a base line and draw it horizontally (this habit will make constructing triangles easier later on). Then we put the protractor on and measure an angle of 61^0 from the start line.

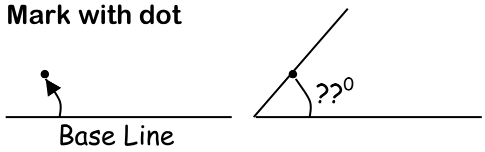


At the 61^0 we draw a dot (probably smaller than the one drawn here, which is larger to make it clearly visible). Finally we remove the protractor and join up the vertex and the dot.



To summarise how to construct an angle…

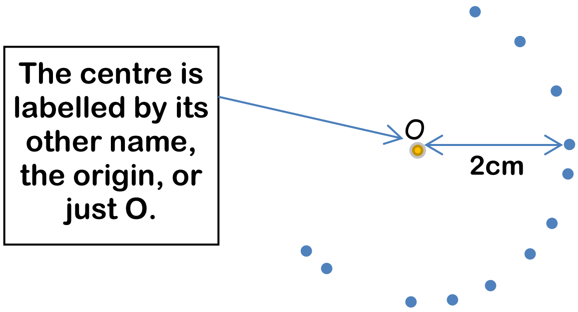
1) Draw Base Line
2) Measure Angle
3) Mark with dot
4) Draw 2^{nd} Side
5) Label Angle



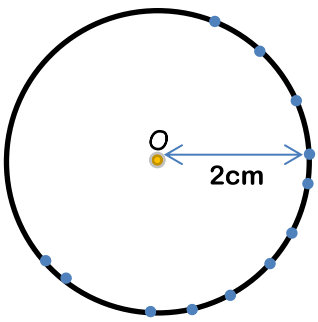
Step 3) Constructing Circles

A circle has two important features, it's centre and its radius. The centre gives it a position, and the radius (the distance from the centre to the edge, called the circumference) gives it its size.

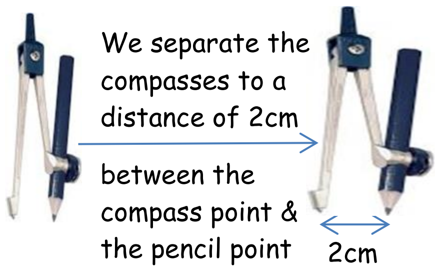
Let's try and construct a circle of radius 2cm. So every point on the circumference (this is a special name for the edge of a circle) is 2cm from the centre. Let's draw a few points that are 5cm from the centre.



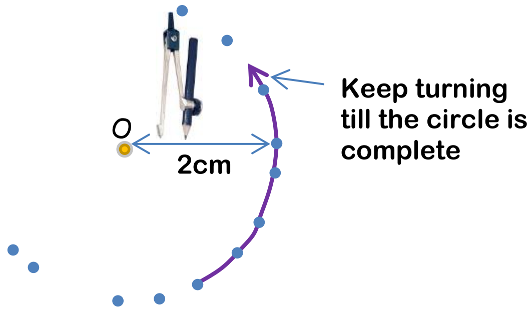
You can see that all these points that are exactly 2cm from the centre (or origin) are forming what we know is a circle. Let's fill the shape in with a curved line…



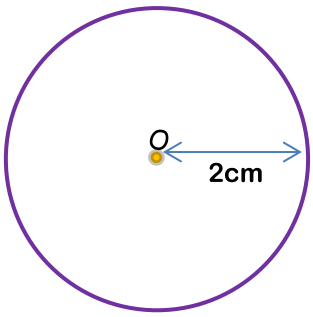
But the computer drew this circle for us, when constructing on paper we need to use a tool called a pair of compasses.



Then putting the compass point on the centre of the circle, we turn the pencil around the centre drawing a line as we go.

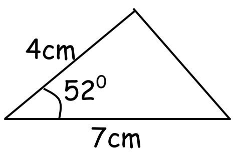


It should look like this.



Step 4) Constructing Triangles

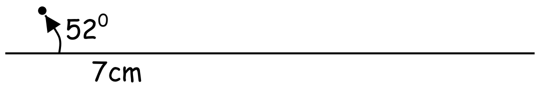
Let's try and construct a triangle from a sketch.



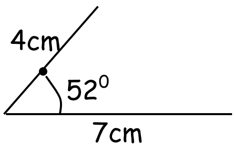
This is like constructing an angle of 52^0 but here the sides we turn from and towards have to be specific lengths. We start facing along the 7cm side, and turn through 52^0 to face along a 4cm side.

Firstly we draw the base of 7cm.

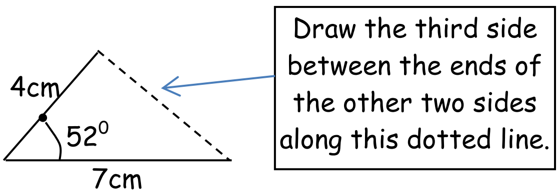
Then we measure the 52^0 angle and mark a dot just like we are constructing a 52^0 angle.



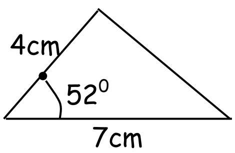
Now when we construct the other side, we have to make it exactly 4cm long by placing the zero mark of the ruler at the vertex (at the left end of the line) and lining this up with the dot and stopping the line at 4cm.



Then we just have to join the ends of the two lines to create the third side.

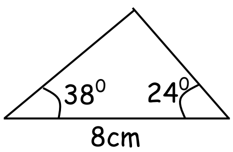


So the triangle looks like this....

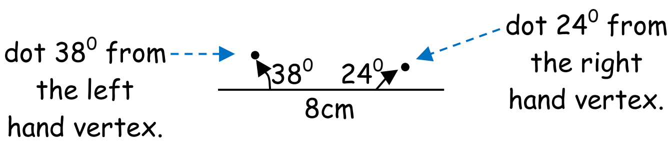


We call this a "Side, Angle, Side," triangle construction, because these were the pieces of information we were given about the triangle; two sides and the angle between them.

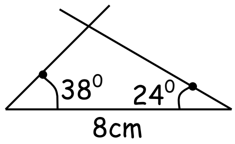
Let's look at a "Angle, Side, Angle," triangle construction where we are given two angles at either end of a given side. Here's a sketch.



Firstly we construct the given base side. Then we measure the angles at either end and draw a dot.

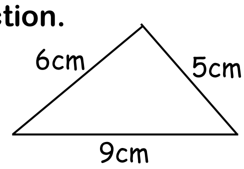


We don't know how long the two sides are, but we know their direction, so we line them up with either end of the base, and the dots, and draw them extra-long.

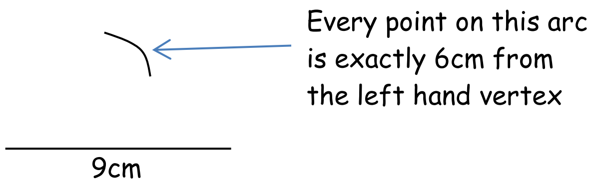


You can see we have the correct triangle, with all sides and angles as labelled, plus two little tails at the top of 2 of the sides. It is normal to leave these on the construction. We call them construction lines. They are not part of the shape we are constructing, but show how we constructed it.

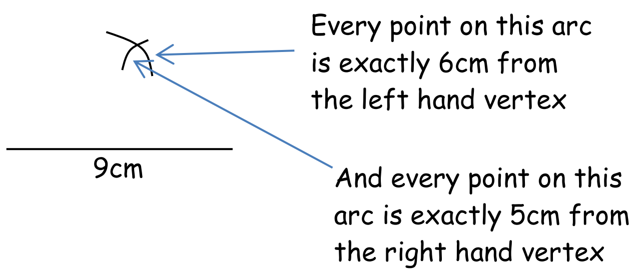
Finally we'll look at triangles where all 3 sides are given but no angles, called a "Side, Side Side," triangle construction.



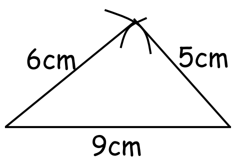
We need to find the vertex where the 6cm and 5cm sides meet. After constructing the 9cm base, we use a pair of compasses to draw an arc of several points that are 6cm from the left hand vertex.



Then we construct a second arc of points exactly 5cm from the right hand vertex.



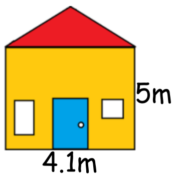
The point where these two arcs cross is exactly 5cm from the right hand vertex and 6cm from the left hand vertex, in other words it is the vertex where the 6cm and 5cm sides meet. So now we fill in the two sides (leaving the construction arcs as for (angle, side, angle).



Step 5) Scale Drawings.

A builder is planning to paint the front of his house and needs to work out how much paint to use. She wants to make a scale drawing of the front face using a scale of 1cm : 1m. She hasn't measured the doors or windows, should she? Either way can you create the scale drawing she wants?

Here is a sketch…



The scale drawing needs to be a rectangle, the roof and doors cannot be added as we don't have accurate info for them.

It's height can be found by scaling up from

1cm : 1m (multiply by 5)

5cm : 5m

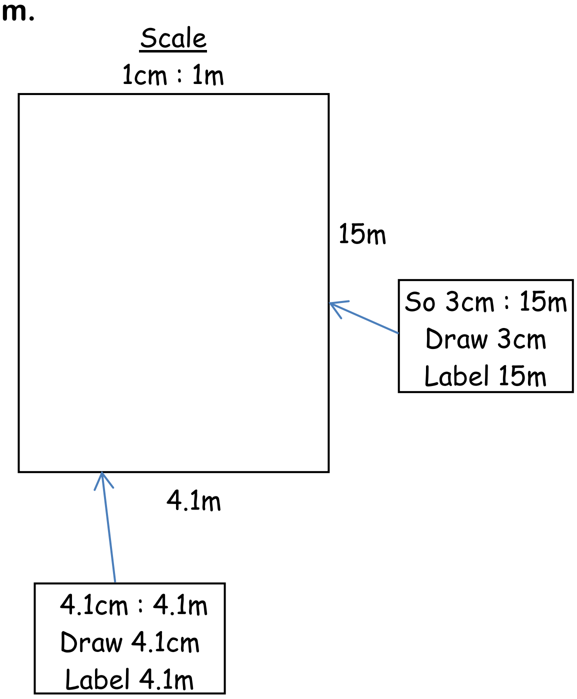
So the height will be 5cm (but labelled 5m!)

The width can be found similarly

1cm : 1m (multiply by 4.1)

4.1cm : 4.1m

So the width of the scale drawing will be 4.1 cm.

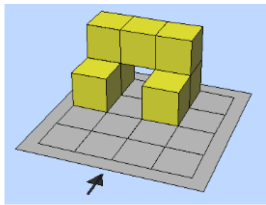


It is very important that we construct this 4.1cm wide, 5cm high rectangle exactly.

We can then use it for other calculations. For example if we needed to know the diagonal length from corner to corner we can measure it in cm, and convert this back to m.

Step 6) Plans & Elevations.

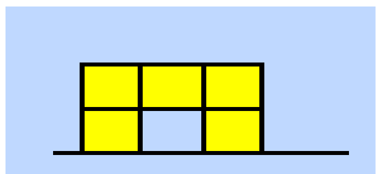
Have a look at this 3D shape.



We can make 2D representations of how the shape will look from the top, the front or the sides. A 2D view from the top of a shape we call a plan. A 2D view from the front or back or sides we call an elevation. Of course we have to define what the front is first, done on the above drawing with the black arrow. The grey base also helps us see what the bottom side of the shape is (opposite the top side, where the plan view is from).

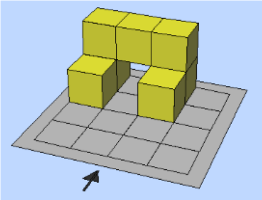
First let's try and make a front elevation. You can divide the view into 3 columns, each two squares high. The left and right have two yellow blocks in place, but the middle column has a gap right through the shape (when looking from the front) at the bottom, and a yellow block on top. So it looks like this.

A Front Elevation



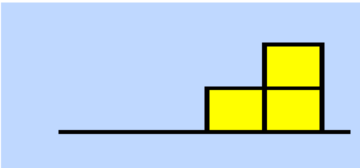
It is interesting to note that the two yellow squares on the bottom layer of this front elevation actually represent yellow blocks that are 2 blocks deep, but they look exactly the same as the 3 yellow swaures on the top row, which each represent only one yellow block. One block will make the front look yellow, just a 2 blocks deep will show a yellow front face (or 3 blocks deep from the side elevation). So with an elevation such as these we either can see all the way through (like the bottle middle square) and we don't draw anything, or there is at least one block and we can see only the front blocks front face.

Now let's make a side elevation from the right hand side.

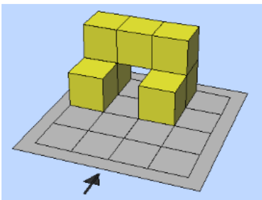


From the right there are only two columns> the right hand side is two blocks high so we'd draw the two yellow squares of the front block front face (these ones are 3 blocks deep, but we still just get 1 yellow square) The left hand column has a yellow block at the bottom but not one at the top. There are no blocks in that top left position all the way through the shape, as we look from the right.

A Right Side Elevation

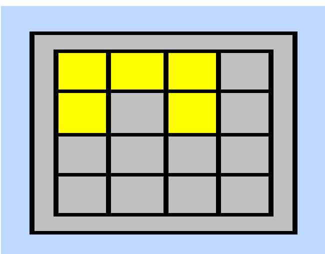


Finally let's look from the top. You might want to call it a top elevation, but it has a special name, a plan view. One word for a map in French is "un plan," and plans or maps always show a scale view of something from directly above!



We can look at this as two rows. The top row has three yellow squares (even though the middle one is only one block deep, it still leaves us seeing just it's front face). The bottom row will only have squares on the left and right as the middle has a gap all the way through the shape (looking from above).

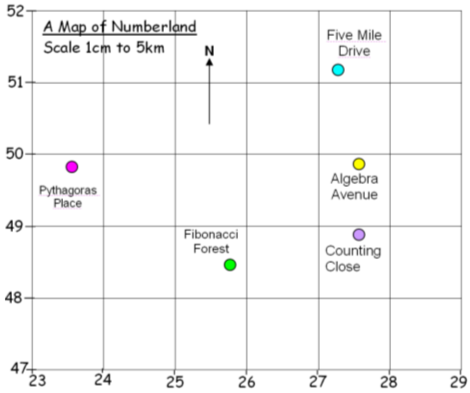
A Plan



Step 7) Maps: Scales, Grid References & Bearings

A map is a type of scale drawing for showing an area we might want to know better or find our way around, such as a town, an area of countryside or a whole continent.

Here is a map of Numberland.



Now clearly the drawing is not a real construction because a whole land must be bigger than this piece of paper. But a map is type of scale drawing like those we looked at in step 4.

If we wanted to know the distance between Pythagoras Place and Five Mile Drive we would first measure them on the map. Let's say their map dots are 8cm apart.

The scale of this map is

1cm : 5km (times both by 8 to get 8cm)

8cm : 40km

So the real life distance between Pythagoras Place and Five Mile Drive is 40km.

Note: "real life," is said with poetic licence, these places only exist in yogi Toby's head, and perhaps now your head too!

The little numbers at the end of each grid line are used to help us pin point different places on the map. Just like with coordinates on a grid we always list the left and right horizontal (x-axis) coordinates first, and the up and down vertical (y-axis) coordinates second.

Here are the grid references of some of the places on the map.

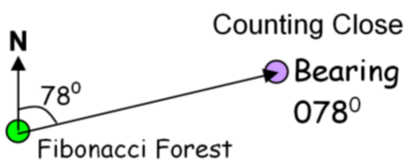
Fibonacci Forest is at 2548
Counting Close at 2748
Algebra Avenue at 2749

These are called four figure grid references. They give as a vertex and tell us the place is in the square top right of that vertex. However a 6-figure grid reference gives us a bit more accuracy, but we have to imagine the square broken into 10 more sections.

So for example because Fibonacci Forest is horizontally $\frac{8}{10}$ of the way through square 2548 we represent this horizontal position as 258. And it is about ½ way vertically up through square 2548 which is $\frac{5}{10}$ so we call this vertical position 485. So overall the 6-figure grid reference for Fibonacci Forest is at 258485 Similarly Counting Close is at 276489 and Algebra Avenue at 276498.

Finally let's look at 3-figure bearings. A bearing is a measure of the direction from one place to another, using an angle clockwise from North.

Let's look at the bearing of Counting Close from Fibonacci Forest. First we draw a vertical line upwards for North (a bearing of 000^0). And we draw a line between the two places.



Then we measure the angle of turn clockwise from facing North at Fibonacci Forest to facing Counting Close. It is 78^0.

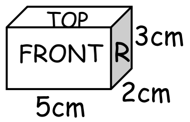
However our bearing is always given as three figures. This prevents confusing things like 71^0 and 710^0 as the former would be written as 071^0.

So the bearing from Fibonacci Forest to facing Counting Close is 078^0.

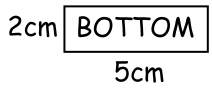
Step 8) Constructing Nets of 3D Shapes

The net of a 3D shape is a 2D shape that can be folder up to make the 3D shape it represents. Constructing these is a skill linked to plans and elevations (Step 5). However now we have the added challenge of not just seeing the shape from each side, but working out where each face goes when we flatten out, or unfold, the 3D shape.

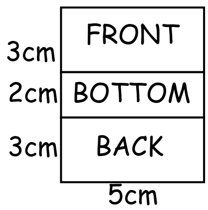
Let's start by constructing the net of a cuboid. This means that all the faces we construct will be rectangles.



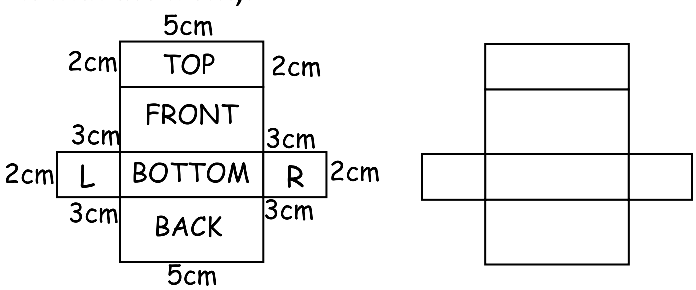
First let's construct the bottom rectangle. It is 5cm wide and 2cm deep.



Now we can put the front and back rectangles either side of it. They both join the bottom along one of their 5cm faces.

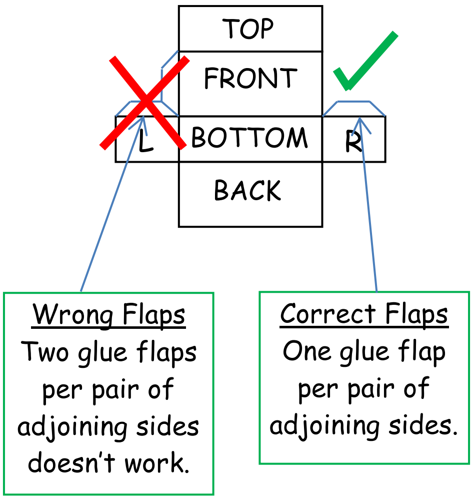


Then we can add the left and right faces either side, attached along their 2cm sides. The top face can be attached, along it's 5cm side either to the front or back (here we'll put it with the front).

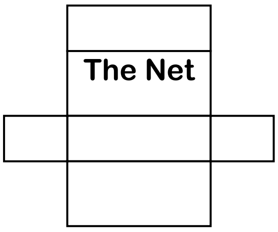


So we have constructed the net of the above cuboid. The labels aren't actually part of the net, so the shape on the right is the final net, the left is to help you see how it fits together.

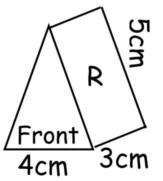
Now we can fold it up to make the 3D shape but unfortunately it won't hold together, if you let go it flaps outward. To solve this we can add some flaps to our net that we can use to glue the 3D shape together. We need just one flap for each pair of adjoining sides (otherwise the tow flaps will get in each other's way at the sticking stage).



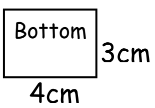
Don't confuse the glue flaps as part of the net, they are an addition to the net to allow you to glue the net into a 3D shape if you want to, but the net itself is just this (without any flaps!...



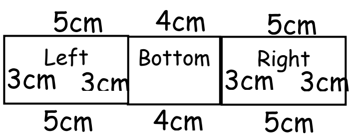
Let's try to make the net of one more 3D shape that is not a cuboid. This is a triangular prism.



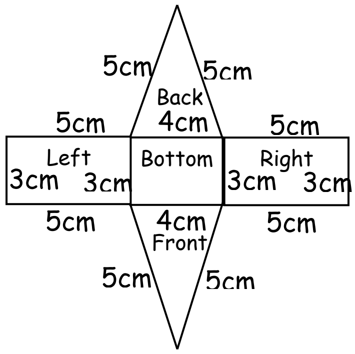
First we construct the bottom face, which is 4cm wide and 3cm deep.



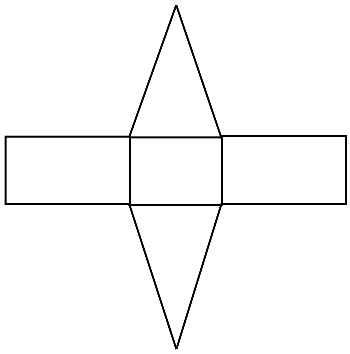
Then we add the right and left faces which adjoin with the 3cm sides of the bottom. The meet along the top edge of the 3D shape.



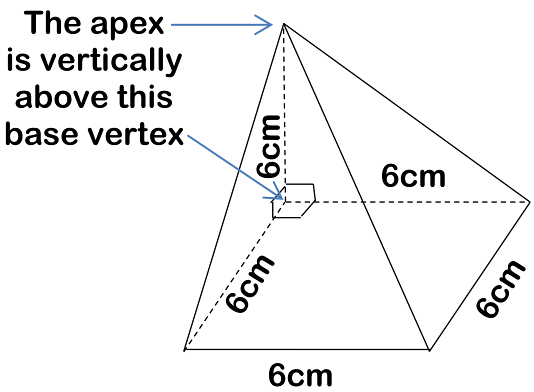
And finally we can add the front and back triangular faces.



The labels are not part of the net so here is how it looks without them.



We will end this step with a challenge. With two friends, each construct a (6cm) square based pyramids, where the apex is (6cm) vertically above one of the sqaures vertices.



Can you fit your three pyramids together to make a (6cm) cube? If you want to relate this to some advancved size steps go to step 16 of size!

9) Constructing Line (Perpendicular) & Angle Bisectors

To bisect simply means to cut into two equal pieces.

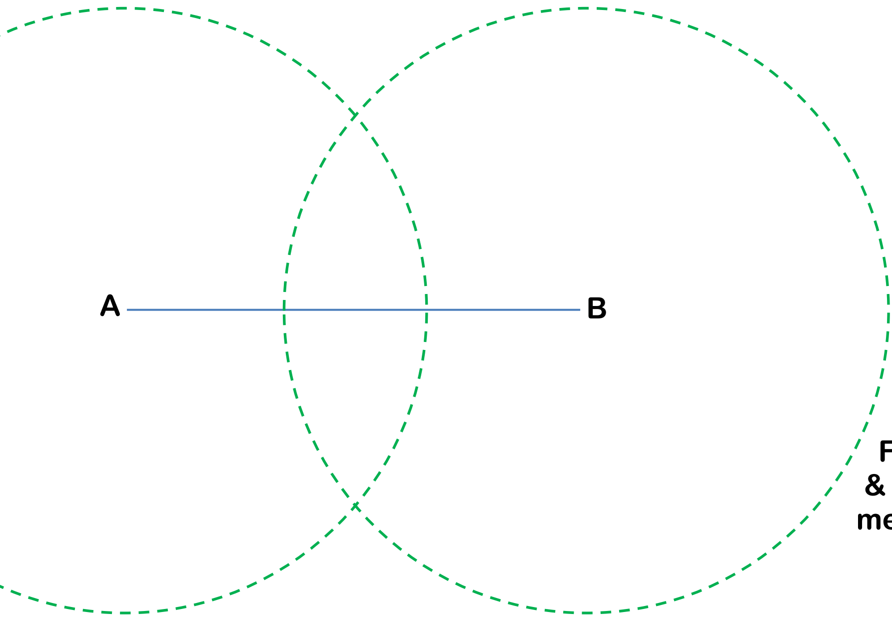
Bi - sect = 2 - sections.

When we cut a line in half, we also create a right angle, as the line that cuts it is perpendicular (at right angles) to it. Hence a line bisector's formal name is a perpendicular bisector.

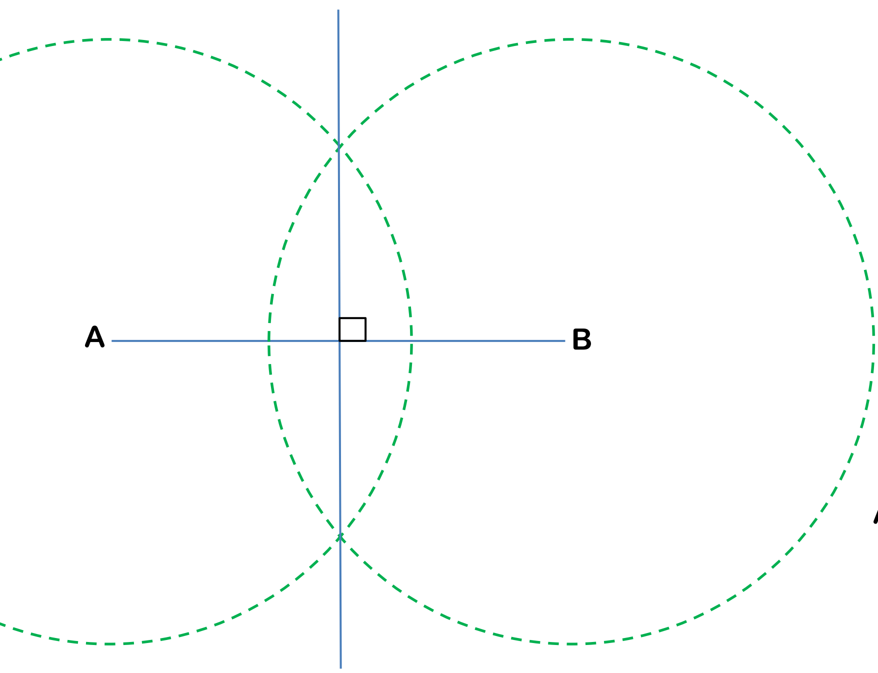
Let’s try and bisect a line ab that is 6cm long.

We do this by finding a point above and below the line that is equidistant (an equal distance) from both A and B. We can do this by drawing two intersecting congruent (same sized) circles, with centres A and B.

These two circles, though they are congruent (of the same size) do not intersect. We have to make the radius of the circles large enough so they go more than half way across the line ab.

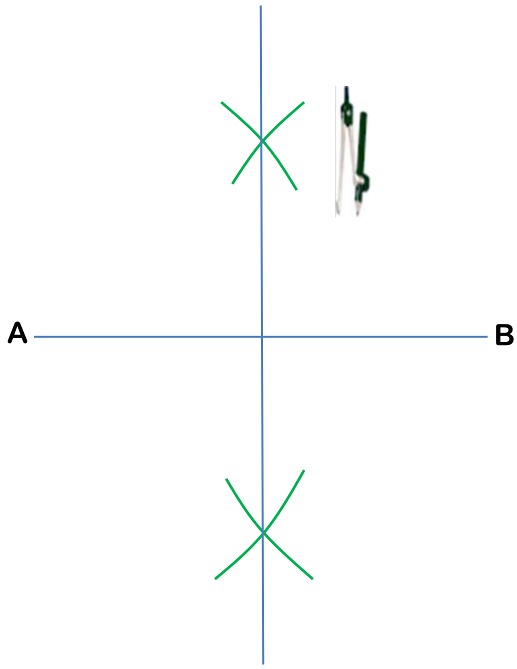


The two points where the lines cross are equidistant from A and B, let’s join them up!

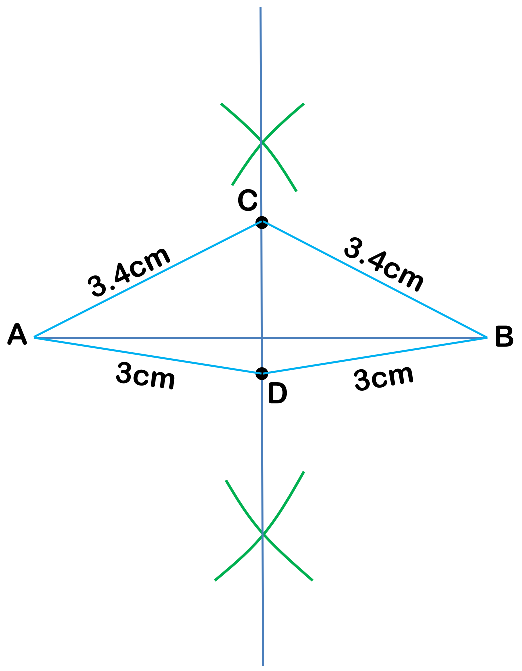


Firstly measure the two sides and see that they are the same (they should both be 3cm). Then measure the angle and see that it is a right angle.

The two circles take up a lot of space! We only need the part of the circles near where they cross, just enough to make sure the second curve crosses through it. So we can use just the small arc of each circle, above and below AB, to show us where they cross. So our actual line bisection will look as below.



Finally let’s take a couple of other points C & D) on our perpendicular bisector line and measure how far they are from both A and B.



AC = BC = 3.4cm   and   AD = BD = 3cm

In fact any point on the line bisector is equidistant from A and B. It can be called a line of equidistant.

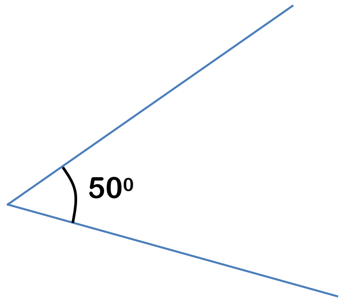
In a sense this technique is a 3 for the price of one! We get a line bisector, a perpendicular bisector and a line of equidistance.

The line bisector is useful if we simply want to cut a line into two equal parts.

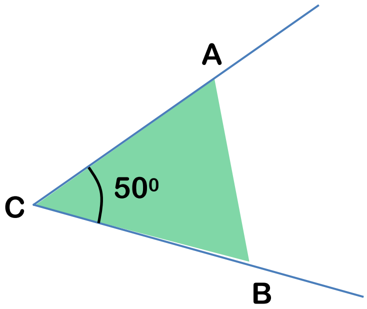
The perpendicular bisector is useful if we want to construct a right angle.

The line of equidistance is useful if we want the line of all points an equal distance from two other points; as we often do in the next topic called loci…

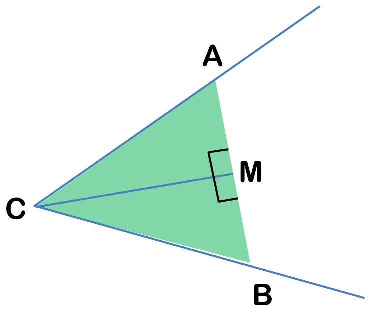
Secondly we need to learn how to bisect an angle, that is to cut it exactly in half into two equal parts. Let's work with a 50^0 angle (these sketches are not to scale).



Let's put an isosceles triangle with the apex at the 50^0 angle (c), and label the two vertices along its base A and B.

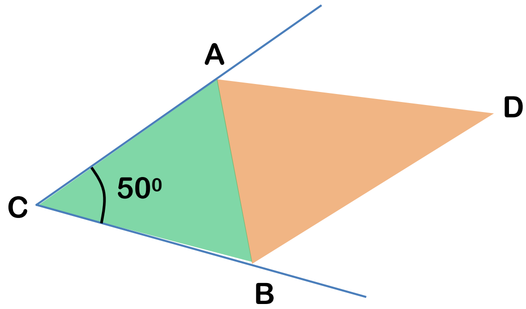


Let's look at a line joining C with the midpoint of AB. The midpoint M of AB, we can see that the apex of the second (D) bisects the angle.

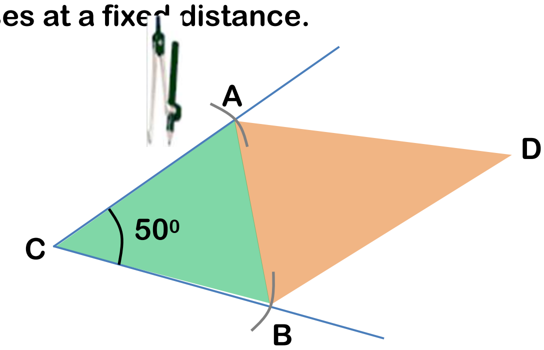


angles ABC and BAC are equal because of the isosceles triangle. And the angles AMC and AMB are equal, as they are both right angles created by the angle bisection. So that means angles ACM and BCM must be equal, which is what we want!

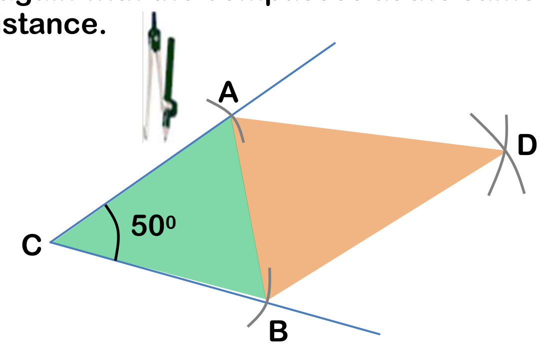
We can create this line bisector for AB by adding another isosceles triangle ABD, and joining CD.



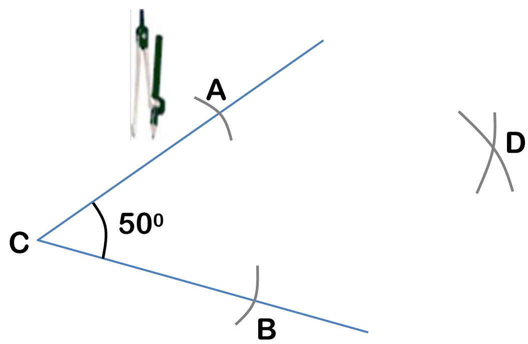
We don't actually need the isosceles triangles, just the points A, B and D. The arcs at A and B are constructed with the compasses at a fixed distance.



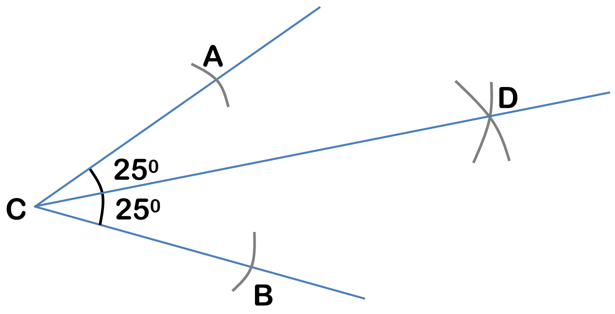
The arcs at D (from A and B as centres) are drawn again with the compasses at the same fixed distance.



The isosceles triangles that helped us understand how this works do not need to be coloured in, or even drawn or each time.



Then finally we join up C and D.



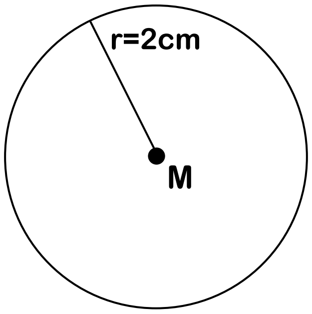
This line CD exactly bisects the angle at C, and is known as an angle bisector.

Step 10) Loci

A locus (loci is the plural) is a set of points that follow a rule or group of rules. For example the rule could be any point on a solid object in the interior of your house. In this case to colour in all the points in this loci, you'd have to colour in all the walls and all the furniture inside your home!

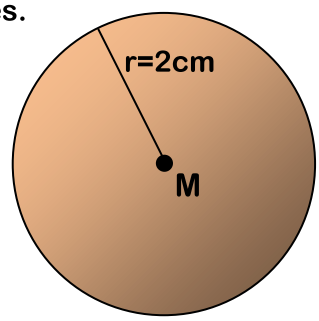
A good starting point is rules that give a locus of points a fixed distance from a given point.

Let's construct the locus of points exactly 2cm from a point M.

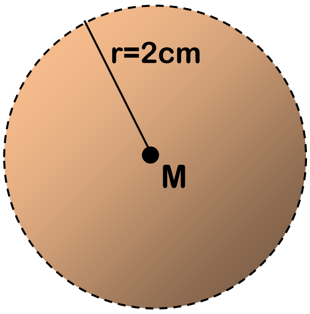


This will simply make a circle of radius 2cm, because every point on a circle, is its radius distance from the centre. Re-read the first half of step 3 if you want to understand this more deeply.

Now what should we do if we want the locus of points less than or equal to 2cm from m. This includes every point on the line we drew before, but also includes all the points inside the circle (distances like 1cm, 1.7cm, 0.2cm from m). We simply shade the shape to represent the inclusion of this space within the loci rules.



But there is one more possibility here. What if we want the locus of points less than 2cm from m (but not equal to 2cm)? We have a neat trick for this. We simply use a dashed line on the diagram to divide regions, where the line itself isn't included.



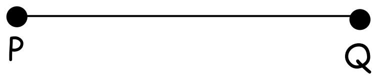
This is the same technique we use for drawing graphs of regions using inequalities.

For $y = 2x + 3$ we just draw a solid line as every point on the line represents this equation.

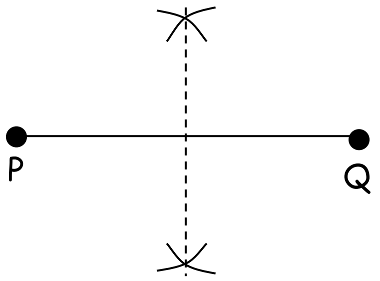
And for $y \geq 2x + 3$ we draw a solid line and shade the region above it as every point on the line and all the points above it represent the inequality.

But for $y > 2x + 3$ we use a dashed line to state that the line divides to regions, but the line itself isn't included. Then we shade in the region above the line.

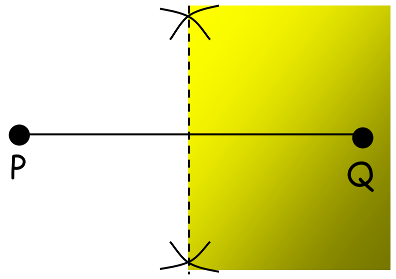
Now lets look at finding the locus of points closer to P than Q on the following drawing.



We can construct a line bisector to give all the points an equal distance from P and Q. Then the points closer to Q will be on the right of it, and the points closer to P will be on the left.

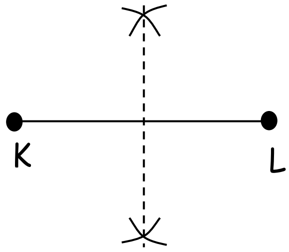


We have to use a dashed line because the line itself is not included (the wording was closer to, not closer or of equal distance). Finally we just shade the region on the right.

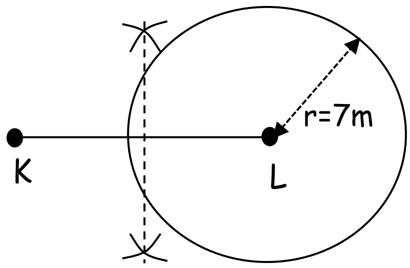


Finally let's try a more complex loci with multiple rules. Shade the locus of points closer to K than L & closer than or exactly 7m from L .

Firstly we form a perpendicular bisector, to show the points that are closer to K than L. We need to use a dashed line, as it says closer to (not closer or equal to K) so the line itself is not included in the locus.



The region closer than or exactly 7m from L will be a circle of diameter 7m (to the scale of your diagram). It will need to be a solid line, as the line is included as it says "or equal to 7m from L."



Finally the region we need to shade is that which is both on the left of the dotted straight line, and within the solid circle.