c) Understanding Fractions.

c) Fractions

Step 1) Unit Fractions; Equal Parts of Wholes

Fractions are parts of wholes. It is where we split a whole (pizza, apple, amount of money, or in this case a rectangle) into a number of equal parts.

1 Whole (rectangle)

If you cut the whole rectangle into 2 equal parts then each part is called a half, written $\frac{1}{2}$.

$\frac{1}{2}$

In the same way, if you divide a whole into 3 equal parts, then each part is called a third, written $\frac{1}{3}$.

$\frac{1}{3}$

And if you share the whole into 4 equal parts, then each part is called a quarter, written $\frac{1}{4}$.

$\frac{1}{4}$

These first three have special names, half, third, & quarter. Beyond this, the names are just the number of parts with "th" on the end (some with odd spellings, e.g. ninth misses the "e").
For example, a sixth is $\frac{1}{6}$.

$\frac{1}{6}$

There is no end to this; you can cut it into huuuge numbers of parts. If you cut the whole into 12 equal parts each part is called a twelfth, written $\frac{1}{12}$.

Or 273 parts is called a two-hundred-and-seventy-third, written $\frac{1}{273}$ (too small to see).

(This is actually a lot less than 273 parts, but it's the most we could make visible)

"Over" & "Divide"

$\frac{1}{3}$ (one third), can also be called:

1 over 3 (position name) & it means

1 ÷ 3 (1 divided by 3)

The divide symbol looks just like fractions when they are written "over."

Top Dot is 1

$\frac{1}{3} = 1 \div 3$

Bottom Dot is 3

Step 2) Proper Fractions; Tops & Bottoms

The **bottom** number is called the **denominator**, and tells you how many parts to split the whole into.

The **top** number is called the **numerator** and tells you how many of those parts you have.

$\frac{2}{3}$ (you say two thirds, or 2 over 3) is just

$$2 \times \frac{1}{3} = 2 \text{ of } \frac{1}{3}$$

$\frac{1}{3}$	$\frac{1}{3}$	

=

$\frac{2}{3}$	

$\frac{5}{7}$ (you say five sevenths, or 5 over 7) is

$$5 \times \frac{1}{7} = 5 \text{ of } \frac{1}{7}$$

$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$		

=

$\frac{5}{7}$		

If the top & bottom are the same, the value is always the same as the whole.

$$\frac{5}{5} = 5 \times \frac{1}{5} = 5 \text{ of } \frac{1}{5} = 1$$

1 Whole

=

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

=

$\frac{5}{5}$

=

1 Whole

Step 3) Improper Fractions; Big Tops

If the top is bigger than the bottom of the fraction, it is called an improper fraction, and it's value is more than 1 whole.

$$= \frac{5}{3} = \frac{3}{3} + \frac{2}{3} = 1\frac{2}{3}$$

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

&

$\frac{1}{3}$	$\frac{1}{3}$	

=

1

&

$\frac{2}{3}$	

$$2\frac{3}{5} = \frac{5}{5} + \frac{5}{5} + \frac{3}{5} = \frac{13}{5}$$

1
1

&

$\frac{3}{5}$		

=

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

&

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

&

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	

Note that any whole number can be written as a fraction over 1.

$$2 = \frac{2}{1}, \quad 3 = \frac{3}{1}, \quad 4 = \frac{4}{1}, \quad 5 = \frac{5}{1} \ldots$$

Step 4) Adding Fractions (with the Same Bottoms)

To add fractions with the same bottoms, you simply add the tops.

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$		

=

$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$		

=

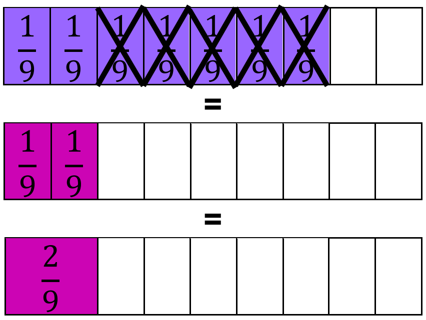
$\frac{5}{7}$		

Step 5) Subtracting Fractions (with the Same Bottoms).

To subtract fractions with the same bottoms, you simply subtract the tops.

$$\frac{7}{9} - \frac{5}{9} = \frac{7-5}{9} = \frac{2}{9}$$

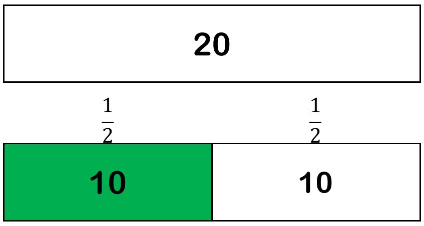
$\frac{7}{9}$		



Step 6) Unit Fractions of Amounts; A Fair Share!

To find a unit fraction of an amount, you split it into equal parts.

To find $\frac{1}{2}$ of 20, you split 20 into 2 equal parts, in other words 20 ÷ 2.



So $\frac{1}{2}$ of 20 = 10. More formally:

$$\frac{1}{2} \text{ of } 20$$

$$= \frac{1}{2} \times 20$$

$$= \frac{20}{2}$$

$$= 20 \div 2$$

$$= 10$$

Similarly, to find $\frac{1}{3}$ of a number, you ÷ 3

To find $\frac{1}{4}$ of a number, you ÷ 4

To find $\frac{1}{5}$ of a number, you ÷ 5

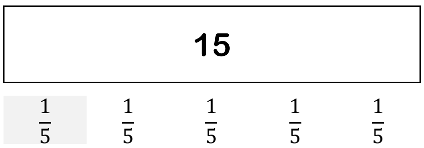
$$\text{So } \frac{1}{5} \text{ of } 15$$

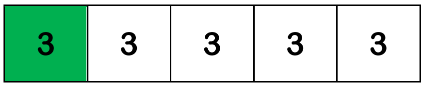
$$= \frac{1}{5} \times 15$$

$$= \frac{15}{5}$$

$$= 15 \div 5$$

$$= 3$$





Step 7) Proper Fractions of Amounts

$$\frac{3}{5} \text{ of } 15$$

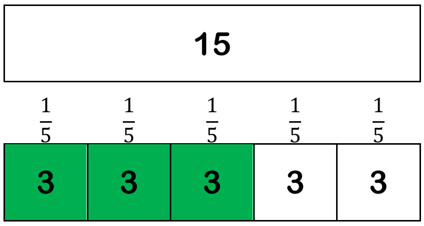
$$= 3 \times (\frac{1}{5} \times 15)$$

$$= 3 \times \frac{15}{5}$$

$$= 3 \times (15 \div 5)$$

$$= 3 \times 3$$

$$= 9$$



So $\frac{3}{5}$ of 15 = 9

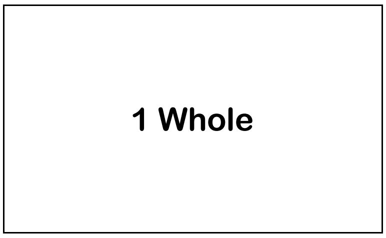
8) Multiplying Unit Fractions (A Fraction of a Fraction)

To multiply unit fractions, you simply multiply the bottoms and the top is still 1

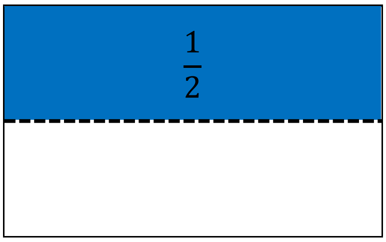
$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{2} \text{ of } \frac{1}{3} = \frac{1}{2\times3} = \frac{1}{6}$$

Let's see why, we'll start with a whole.

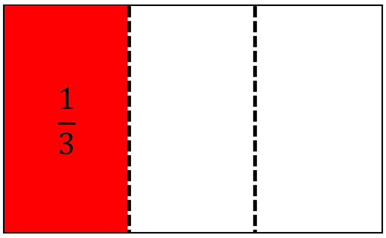
(This works really well using a piece of A4 paper as the whole, & folding it)



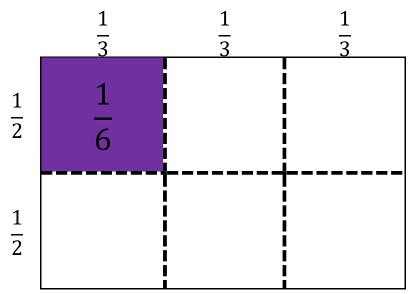
We can cut our whole vertically into 2 to get $\frac{1}{2}$



We can cut our whole horizontally into 3 to get $\frac{1}{3}$



However, we want $\frac{1}{2}$ of $\frac{1}{3}$. Let's cut our whole into $\frac{1}{3}$ horizontally & then cut that $\frac{1}{3}$ in $\frac{1}{2}$ vertically.



You can see that this cuts the whole into 2 x 3 = 6 equal parts, so it is $\frac{1}{6}$.

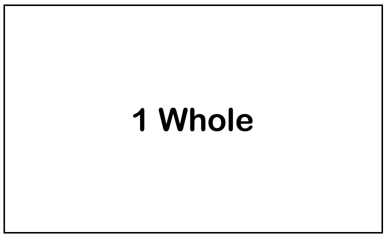
Whatever the bottom of the two fractions, you will always multiply them to give the number of parts of the whole, so we always just multiply the bottom numbers.

9) Multiplying Proper Fractions

To multiply proper fractions, you simply multiply the tops & multiply the bottoms.

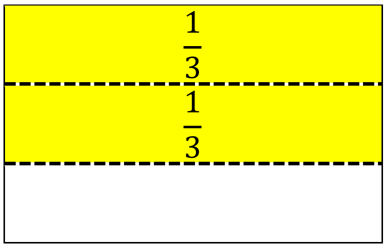
$$\frac{2}{3} \times \frac{3}{4} = \frac{2}{3} \text{ of } \frac{3}{4} = \frac{2\times3}{3\times4} = \frac{6}{12}$$

(Note $\frac{6}{12} = \frac{1}{2}$, we'll see why in step 8)

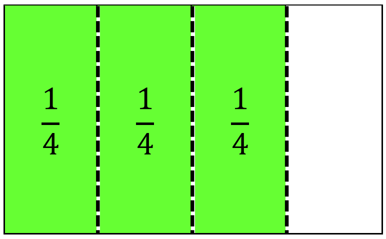


(again it works really folding a piece of A4 paper)

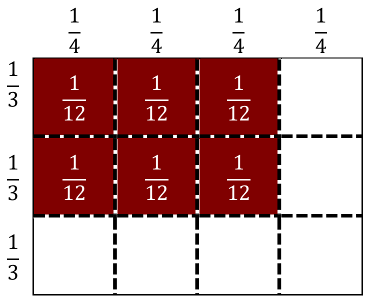
We can cut our whole vertically into 3, and take 2 of them to get $\frac{2}{3}$



We can cut our whole horizontally into 4 and take 3 of them to get $\frac{3}{4}$



But we want $\frac{2}{3}$ of $\frac{3}{4}$. Let's divide our whole into $\frac{3}{4}$ horizontally & then divide that $\frac{3}{4}$ into $\frac{2}{3}$ vertically.



We get a total of 3 x 4 = 12 parts, & can see the total number of parts the whole is cut into is always given by multiplying the bottoms.

Our shaded amount is 2 x 3 = 6 parts, and this will always be found by multiplying the tops.

So we have 6 parts out of 12 parts which is $\frac{1}{12}$.

Step 10) Equivalent Fractions

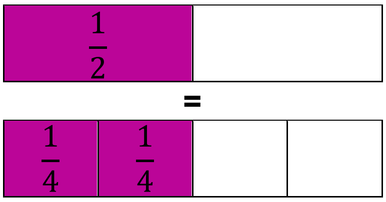
Equivalent fractions represent the same part of the whole. They can be found by multiplying or dividing the top and bottom of a fraction by the same number.

$$\frac{1}{2} \overset{\times 2}{=} \frac{2}{4} \quad \text{and} \quad \frac{2}{3} \overset{\times 3}{=} \frac{6}{9}$$

Similarly

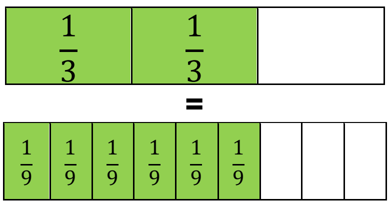
$$\frac{2}{4} \overset{\div 2}{=} \frac{1}{2} \quad \text{and} \quad \frac{6}{9} \overset{\div 3}{=} \frac{2}{3}$$

Let's see that this works in a picture.



When you spit each part in 2, you double the number of parts over all, and you also double the number of shaded parts, so doubling both the top and the bottom.

Similarly $\frac{2}{3} = \frac{6}{9}$



However there is a neater why of understanding this. We know that any fraction with the top and bottom the same is just 1.

$$\frac{1}{2}$$

$$= \frac{1}{2} \times 1$$

$$= \frac{1}{2} \times \frac{2}{2}$$

$$= \frac{2}{4}$$

$$\& \ \frac{6}{9}$$

$$= \frac{6}{9} \div 1$$

$$= \frac{6}{9} \div \frac{3}{3}$$

$$= \frac{2}{3}$$

Step 11) Adding & Subtracting Fractions with Different Bottoms

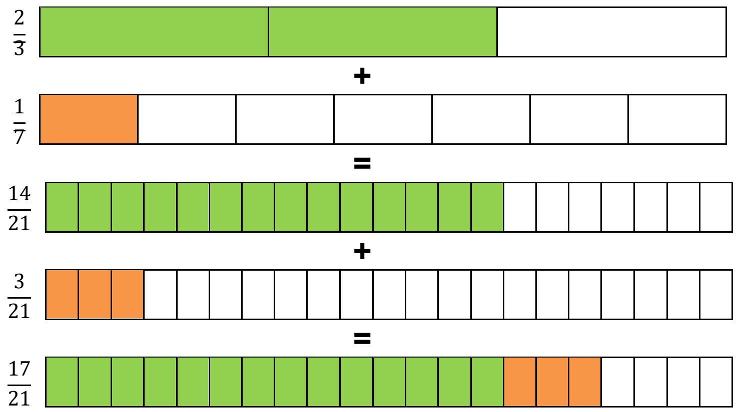
$$\frac{2}{3} + \frac{1}{7}$$

$$= \frac{2}{3} \times \frac{7}{7} + \frac{1}{7} \times \frac{3}{3}$$

$$= \frac{14}{21} + \frac{3}{21}$$

$$= \frac{17}{21}$$

This won't change the fractions' values as we're just x1. $\frac{7}{7} = \frac{3}{3} = 1$



Step 12) Dividing by a Unit Fraction

$$\div \frac{1}{2} = \times 2$$

$$\div \frac{1}{3} = \times 3$$

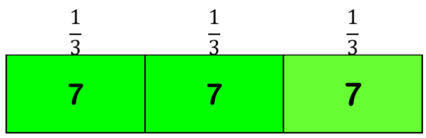
$$\div \frac{1}{4} = \times 4$$

$\div \frac{1}{5} = \times 5$**... and so on**

But why?

Let's look at $7 \div \frac{1}{3}$

You share out 7 to each $\frac{1}{3}$ **of a whole. So how much will the whole get?**



The whole will get 7 x 3 = 21

So $7 \div \frac{1}{3} = 7 \times 3 = 21$

Step 13) Dividing by a proper fraction

Dividing is the opposite of multiplying. So we can simply dived the tops and divide the bottoms.

$$\frac{6}{28} \div \frac{2}{7} = \frac{6 \div 2}{28 \div 7} = \frac{3}{4}$$

If they don't divide nicely, then...

If you divide by $\div \frac{2}{3}$, you have to both $\div 2$ and $\div \frac{1}{3}$

Now as we saw in step 5, $\div 2 = \times \frac{1}{2}$

And we saw in the previous step that $\div \frac{1}{3} = \times 3$

So $\div \frac{2}{3} = \div 2$ and $\div \frac{1}{3} = \times \frac{1}{2}$ and $\times 3 = \times \frac{3}{2}$

In short $\div \frac{2}{3} = \times \frac{3}{2}$

So to divide by a proper fraction, we simply flip the top and bottom and then multiply.

$$30 \div \frac{2}{3} = 30 \times \frac{3}{2} = \frac{30 \times 3}{2} = \frac{90}{2} = 45$$

Step 14) + & - Mixed Numbers

To + or - mixed numbers you either convert to a top heavy (improper) fraction first, or you simply put the '+' sign into the mixed numbers and + or – the whole number and fraction parts separately.

Putting the '+' signs back in (adding):

$$1\frac{1}{3} + 2\frac{2}{5}$$

$$= 1 + 2 + \frac{1}{3} + \frac{2}{5}$$

$$= 3 + \frac{1}{3} \times \frac{5}{5} + \frac{2}{5} \times \frac{3}{3}$$

$$= 3 + \frac{5}{15} + \frac{6}{15}$$

$$= 3 + \frac{11}{15}$$

$$= 3\frac{11}{15}$$

OR converting to improper fraction

$$1\frac{1}{3} + 2\frac{2}{5}$$

$$= \left(\frac{3}{3} + \frac{1}{3}\right) + \left(\frac{10}{5} + \frac{2}{5}\right)$$

$$= \frac{4}{3} + \frac{12}{5}$$

$$= \frac{4}{3} \times \frac{5}{5} + \frac{12}{5} \times \frac{3}{3}$$

$$= \frac{20}{15} + \frac{36}{5}$$

$$= \frac{56}{15}$$

$$= \frac{45}{15} + \frac{11}{15}$$

$$= 3 + \frac{11}{15}$$

$$= 3\frac{11}{15}$$

Putting the '+' sign back in (subtracting):

$$4\frac{2}{3} - 2\frac{3}{4}$$

$$= \left(4 + \frac{2}{3}\right) - \left(2 + \frac{3}{4}\right)$$

$$= 4 + \frac{2}{3} - 2 - \frac{3}{4}$$

$$= (4 - 2) + \left(\frac{2}{3} - \frac{3}{4}\right)$$

$$= 2 + \left(\frac{2}{3} \times \frac{4}{4} - \frac{3}{4} \times \frac{3}{3}\right)$$

$$= 2 + \left(\frac{8}{12} - \frac{9}{12}\right)$$

$$= 2 + \left(-\frac{1}{12}\right)$$

$$= 1\frac{11}{12}$$

Or also converting to an improper fraction would work well.

Improper fraction method usually involves larger calculation, so I slower to do mentally. With subtracting, putting back the '+' sign sometimes leaves you with a negative fraction at the end, which if not desired, leaves with the improper fraction route.

Step 15) × & ÷ Mixed Numbers

The "quick" method is to convert to top heavy fractions:

$$2\frac{1}{2} \times 3\frac{2}{3}$$

$$= \left(\frac{4}{2} + \frac{1}{2}\right) \times \left(\frac{9}{3} + \frac{2}{3}\right)$$

$$= \frac{5}{2} \times \frac{11}{3}$$

$$= \frac{5 \times 11}{2 \times 3}$$

$$= \frac{55}{6}$$

$$= \frac{54}{6} + \frac{1}{6}$$

$$= 9\frac{1}{6}$$

With a divide, you would just flip the second improper fraction and turn to a $\times$ as explained in an earlier step.

This is quite neat, but does not make it easy to understand why this works. For a heavier, but much more understandable method, we can use a grid:

$$2\frac{1}{2} \times 3\frac{2}{3}$$

$$= \left(2 + \frac{1}{2}\right)\left(3 + \frac{2}{3}\right)$$

2	$\frac{1}{2}$	
2 in each row, 3 rows, $2 \times 3 = 6$	**3 in each column, but only $\frac{1}{2}$ a column.** $3 \times \frac{1}{2} = \frac{3}{2}$	3
2 in each row, but only $\frac{2}{3}$ of a row. $2 \times \frac{2}{3} = \frac{4}{3}$	**$\frac{2}{3}$ of sqaure high, and $\frac{1}{2}$ a square wide.** $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$	$\frac{2}{3}$

$$6 + \frac{3}{2} + \frac{4}{3} + \frac{2}{6}$$

$$= 6 + \frac{9}{6} + \frac{8}{6} + \frac{2}{6}$$

$$= 6 + \frac{19}{6}$$

$$= 6 + \frac{18}{6} + \frac{1}{6}$$

$$= 6 + 3 + \frac{1}{6}$$

$$= 9 + \frac{1}{6}$$

$$= 9\frac{1}{6}$$

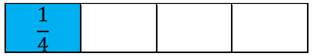
Fraction Facts

Fraction	**Decimal**	**%**
$\frac{1}{2} = \frac{2}{4}$	**0.5**	**50%**
$\frac{1}{4}$	**0.25**	**25%**
$\frac{3}{4}$	**0.75**	**75%**
$\frac{1}{3}$	**0.333…** $= 0.\dot{3}$	$33.\dot{3}\%$ $\approx$**33%**
$\frac{2}{3}$	**0.666…** $= 0.\dot{6}$	$66.\dot{6}\%$ $\approx$**67%**
$\frac{1}{5}$	**0.2**	**20%**
$\frac{1}{8}$	**0.125**	**12.5%**
$\frac{1}{9}$	$0.\dot{1}$	**11%**
$\frac{1}{10}$	**0.1**	**10%**

Super Topic **Fractions** Summary Sheet

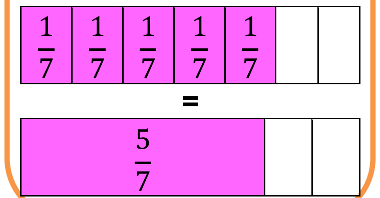
1) Unit Fractions

A quarter, written $\frac{1}{4}$



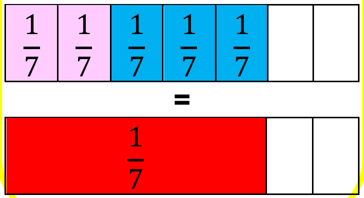
2) Proper Fractions

$\frac{5}{7}$ (you say five sevenths, or 5 over 7) is $5 \times \frac{1}{7} = 5$ of $\frac{1}{7}$



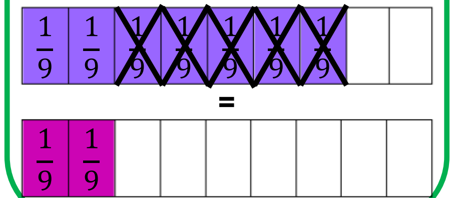
3) + Fractions (Same Bottoms)

$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$



4) – Fractions (Same Bottoms)

$\frac{7}{9} - \frac{5}{9} = \frac{7-5}{9} = \frac{2}{9}$



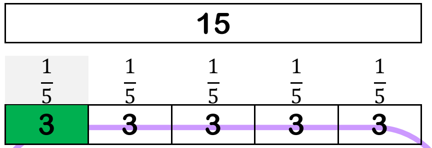
5) Unit Fractions of Amounts

To find $\frac{1}{2}$ of a number, you ÷ 2

To find $\frac{1}{3}$, you ÷ 3,

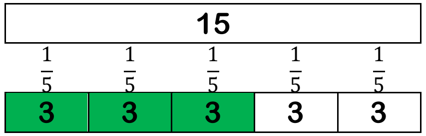
to find $\frac{1}{4}$ of, you ÷ 4…

So $\frac{1}{5}$ of $15 = 15 \div 5 = 3$



6) Proper Fractions of Amounts

$\frac{3}{5}$ of 15



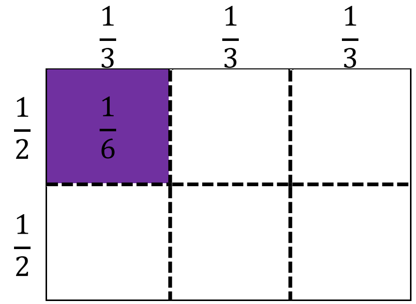
$= 3 \times \left(\frac{1}{5} \times 15\right) = 3 \times (15 \div 5)$

$= 3 \times 3 = 9$

7) x Unit Fractions

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{2}$ of $\frac{1}{3} = \frac{1}{2\times3} = \frac{1}{6}$

Cut the whole into $\frac{1}{3}$ horizontally & then cut that $\frac{1}{3}$ in $\frac{1}{2}$ vertically.

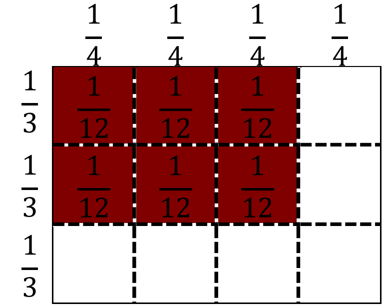


It cuts the whole into 2 x 3 = 6 equal parts, so it is $\frac{1}{6}$.

8) x Proper Fractions

$\frac{2}{3} \times \frac{3}{4} = \frac{2}{3}$ of $\frac{3}{4} = \frac{2\times3}{3\times4} = \frac{6}{12} = \frac{1}{2}$

Divide our whole into $\frac{3}{4}$ horizontally & then divide that $\frac{3}{4}$ into $\frac{2}{3}$ vertically.



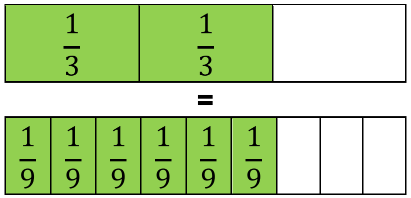
We get a total of 3 x 4 = 12 parts.

aded amount’s 2 x 3 = 6 parts.

o 6 parts out of 12.

9) Equivalent Fractions

$\frac{2}{3} = \frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$



$\frac{6}{9} = \frac{6}{9} \div \frac{3}{3} = \frac{2}{3}$

10) + & - Fractions (Different Bottoms)

$\frac{2}{3} + \frac{1}{7}$ $\boxed{\frac{7}{7} = \frac{3}{3} = 1}$

$= \frac{2}{3} \times \frac{7}{7} + \frac{1}{7} \times \frac{3}{3}$

$= \frac{14}{21} + \frac{3}{21}$

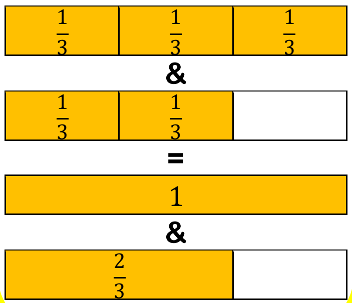
$= \frac{17}{21}$

This won’t change the fractions’ value as we’re just x1.

11) Improper Fractions

If the top is bigger than the bottom, it’s called an improper fraction, it’s value is more than 1.

$= \frac{5}{3} = \frac{3}{3} + \frac{2}{3} = 1\frac{2}{3}$



12) ÷ by a Unit Fraction

$\div \frac{1}{2} = \times 2$, $\div \frac{1}{3} = \times 3$, $\div \frac{1}{4} = \times 4$, $\div \frac{1}{5} = \times 5$, and so on…

13) ÷ by a Proper Fraction

Simply divide the tops & divide the bottoms.

$\frac{6}{28} \div \frac{2}{7} = \frac{6 \div 2}{28 \div 7} = \frac{3}{4}$

If the tops & bottoms don’t divide, we can flip the top and bottom and then multiply.

So $\div \frac{2}{3} = \times \frac{3}{2}$

$30 \div \frac{2}{3} = 30 \times \frac{3}{2} = \frac{30 \times 3}{2}$

$= \frac{90}{2} = 45$

14) + & - Mixed Numbers.

$1\frac{1}{3} + 2\frac{2}{5}$

Split the wholes & fractions!

$= 1 + 2 + \frac{1}{3} + \frac{2}{5}$

$= 3 + \frac{1}{3} \times \frac{5}{5} + \frac{2}{5} \times \frac{3}{3}$

$= 3 + \frac{5}{15} + \frac{6}{15}$

$= 3 + \frac{11}{15}$

$= 3\frac{11}{15}$

15) x & ÷ Mixed Numbers

$2\frac{1}{2} \times 3\frac{2}{3}$

Convert to top heavy fractions.

$= \left(\frac{4}{2} + \frac{1}{2}\right) \times \left(\frac{9}{3} + \frac{2}{3}\right)$

$= \frac{5}{2} \times \frac{11}{3}$

$= \frac{5 \times 11}{2 \times 3} = \frac{55}{6}$

$= \frac{54}{6} + \frac{1}{6} = 9\frac{1}{6}$